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EDITED BY

JOHN R. CLARK, Editor-in-Chief

[illegible] R. SMITH, Associate Editor

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[illegible]ondence relating to editorial matters should be addressed to

JOHN R. CLARK, Editor-in-Chief

[illegible] Yonkers, N. Y., and The Lincoln School of Teachers College, 425 West 123rd Street, New York City

[illegible] relating to subscriptions, advertisement, and other business matters should be addressed to

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THE MATHEMATICS TEACHER

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REPORT OF THE COMMITTEE ON MATHEMATICS IN THE STATE NORMAL SCHOOL CURRICULA PENNSYLVANIA

The Appointment of the Committee. In connection with the notification of the appointment of the committee whose report is about to be given was the following paragraph:

> "At the Lock Haven Conference (of Normal School teachers held in April, 1922) there was a great deal of discussion of the course of study of the different curricula offered by the normal schools. The normal school principals feel that the teachers of the normal schools should have a share in determining the content of the course of study. Because of this point of view, the normal school principals recently voted to have a committee representing them work out a plan by which the teachers would share in determining the content of the course of study. This committee of normal school principals has conferred with subject directors of the Department of Public Instruction and has worked out a plan of procedure. Committees have been appointed for the undertaking of this work in the different groups of subjects of the curricula."

The members of the mathematics committee have been in correspondence during the year, and were in conference at Harrisburg, March 12, 1923, at which time the items covered by this report were agreed upon.

Present Provisions for Mathematics Study. A study of the "Analysis of Distribution of Semester Hours" in the present curricula of our normal schools shows that for Group One not even one hour is given to arithmetic or the teaching of arithmetic as against thirteen hours given to English, seven hours to music, seven hours to art and twenty-seven hours to education. In Groups Two and Four, we find that out of a total of eighty hours, but three hours are assigned to mathematics, which are given over to the teaching of arithmetic.

Varying Qualifications of Entering Students. The first situation to which this committee gave serious consideration is the

qualifications in arithmetic of the hundreds of young men and women who enter the normal schools of the state at the beginning of each school year. These qualifications are so varied that, were these persons classified in this branch, their median ranking would be no higher than sixth grade. Many of them are wanting not only in power to apply the processes of arithmetic to common sense medium grade problems chosen from fields with which seventh and eighth grade pupils should be familiar, but alike deficient in the processes themselves. This statement can be verified by any one who, by one means or another, has undertaken to acquaint himself with the qualifications of these newcomers. For example:

> Of *fifty-three* first year normal school students who attempted to find the quotient of 432.48 divided by .048, *thirty-two* failed to obtain the result; *thirty-eight* of them could not express 4.8% as a decimal, and *thirty-six* could not express .0175 as a per cent. *Thirty-eight* of them could not find the cost of 1625 pounds of coal at $15.25 per 2000 pounds. Of *seventy-five* who were given the Courtis Standard Test in Addition, *twenty-five* did not reach the standard in accuracy set for *fifth grade; forty-two* fell below the standard for the *sixth grade.* In multiplication, *twenty-three* did not exceed the standard set for *fifth grade.*

This varying degree of proficiency in arithmetic might well be expected in these applicants for admission to the normal schools. In most instances they have not received any instruction in this subject since they left the eighth grade; and their training to that point, while excellent in some cases, was, in many others, so unsatisfactory and inadequate as not only to leave them in a state of uncertainty in dealing with processes and helpless in applying them, but, in addition, to engender in them a genuine dislike for the subject. In consequence, the members of classes organized for courses in the teaching of arithmetic in Group Two and Rural Group, second semester, vary in qualifications to such a degree that a considerable portion of them are held back, retarded, while teachers strive to bring the other members to a degree of proficiency such as will justify giving to the work the professional aspect that it should take on from the beginning.

It is no solution of this problem to assert that it is not the function of the normal school to provide for making up deficiencies, and that it must be assumed that this preliminary

work has been done prior to the student's enrollment therein. The fact remains that these persons lacking proper qualifications are with us in great numbers at the beginning of each year, are with us on our own terms, that is, on certificate, so that theorizing on what the schools from which they came should have done does not remove the deficiency. Neither can the problem be brushed aside by saying that those weak in subject matter must make up the deficiency themselves. The fact is that a great majority of these are unable to appraise their own deficiencies, and besides have not the resources to be their own instructors. They must be guided; and, not only guided, but taught. Besides they cannot carry on, with any degree of profit, a course in methods of teaching a subject along with a course in the study of the matter in which they are markedly deficient.

The first recomendation of the committee is, therefore, *that the normal schools of the state ascertain what students on entering are below the standard of the eighth grade in arithmetic and provide not less than a three-hour course for these persons for the first semester.* The committee finds that in the New York normal schools tests are given to ascertain what the incoming student's knowledge is of this subject; and, in addition, there is provided a four-hour course in arithmetic for all groups in the first semester. It also has learned that in the New Jersey normal schools entrance examinations are given in the subject.

Provision for Group One. The course of study provides no arithmetic for Group One. This is an omission so serious that the committee wishes to recommend most earnestly *that for those of Group One who are not preparing specifically to become kindergarten teachers a three-hour course be provided in the second semester in the teaching of arithmetic.*

These prospective teachers of first, second, third grades, being years away from the time at which they themselves were pupils in these grades have but the vaguest notion of either the content of the arithmetic taught therein, or the nice relationships that must be sought out and observed in presenting the first steps in number work. As the course is today, we are sending out a body of teachers practically untrained in presenting number work to the pupils of the grades mentioned, the very grades in which we should find the most masterly presentation of number work. Although these teachers, trained as we are training them

now in group one, will carry with them our diploma, nevertheless they have little more specific qualifications for teaching number work in the first three grades than they had when they entered the normal schools. Many of them, as pupils, were indifferent to the subject, or harbored a dislike for it, and the normal schools are contributing little which will put this group in the right attitude towards number work. It is difficult to understand the basis for the presumption that because these persons have taken courses in English fundamentals, introduction to teaching, oral expression, nature study, music, etc., therefore, they are qualified to teach *number work*. As well might a medical school certify to the fact that a man, who has pursued a specific course on diseases of the eye, nose and throat, thereby becomes a specialist on diseases of the heart and send him forth to claim his victims.

But few primary arithmetics provide the necessary material for the first two grades and organize it. The burden of doing this falls upon the teacher. Authors undoubtedly regard the first three years of number work as the most difficult to handle. How much more will these groups of untrained young women find it so? Yet, we are sending them out without so much as calling their attention to the fact that here is a problem which they must meet. Without giving the matter due consideration some one may say that number work is not taught to any great extent in the first two years. We reply that we find it taught in the first year in one hundred thirteen (113) cities of the one hundred forty-six (146) whose courses were investigated. We find it taught in the second year in one hundred forty (140) of these one hundred forty-six (146) cities; besides, the course of study for Pennsylvania schools provides for number work beginning with the first school year. Further, it was formerly believed that the mastery of the elementary sums, differences, products and quotients is easy of accomplishment, but recent intensive investigations show this view to be erroneous. Within recent years, the whole field of elementary number work has been thoroughly studied, with the result that the best methods of utilizing the fund of unorganized arithmetical knowledge which the child possesses on entering school is set forth; the inherent difficulties encountered in early number work are pointed out; the limits set for the proper amount of subject-matter in each

process for each grade are defined; the number of possible basic sums, differences, products and quotients are listed and the relatively difficult ones set forth for specific study; the necessary and sufficient amount of development work in each new process is carefully set forth; the place of objective representation and such artificial aids as dramatization of nursery rhymes and the acting out of make-believe situations are suggested; the amount of drill work and application that must be given to reach the end sought are carefully outlined for the teacher's use. *All this suggests a body of trained teachers to appreciate and utilize the results of these labors.* It may be said that the necessary preparation may be given in the course listed as The Teaching of Primary Subjects, third semester. We reply that the preparation of the teacher for the task set forth is the work of the *specialist* and not that of the *general practitioner.* To repeat, the committee asks *that three hours be given to the teaching of arithmetic in the second semester for those of Group One who are not confining their work to the sub-primary field.*

Increased Provision for Group Two. The third recommendation of the committee is *that four hours instead of three hours be given to the teaching of arithmetic for Group Two, second semester.* The teachers of the fourth, fifth and sixth grades are those on whom the duty devolves of doing their work in such a masterly way as will not only insure that the pupils attain standards in processes with integers, fractions and decimals set forth for their grades, but also of choosing with great care the problem material to be used to the end that appeal may be made to interests associated with the child in relation to himself, the school, the home and the community. Not only this, but there must be in charge teachers who can create in the pupils of these grades a desire for more knowledge of the subject pursued instead of, as is too often the case, create in him a dislike for it.

This task is the task of the master teacher; the teacher, as Fitch points out, who knows her subject; the teacher who knows not only the problems of her own group, but whose perspective carries her back through the primary grade program and up to that of the upper grades; the teacher who possesses a sure mastery over a relatively extensive field. She cannot become a specialist in teaching a subject in a certain few courses unless she can intelligently utilize what has already been done and shape

her work for dove-tailing into what is to follow. Four hours per week for one semester is required to prepare such teachers of arithmetic for this group. Subject-matter must be taught intensively; subject-matter with method; not subject-matter in the desultory way in which too many were taught it, but subject-matter presented in a way that will serve as the teacher's model when she comes to give it back to her pupils.

As applicable to the courses for the second group and rural group the committee is entirely in accord with the following from the Bulletin No. 14, pages 226 and 231, "The Carnegie Foundation for the Advancement of Teaching":

> "One may hazard an opinion that courses that emphasize careful study of subject-matter to be taught and the best method of presenting it to elementary pupils are those that have the largest practical value." And again: "The large problem of organizing the subject-matter for teaching and of indicating the points at which the teacher's emphasis must fall can in general be solved nowhere so well as in the subject-matter itself. Whether it be a 'review' or a 'new view' the student's experiences in learning or re-learning will form the best concrete basis for an understanding of the special 'pedagogy' of the subject. While these experiences are fresh, they should be studied and discussed to the end that they may be registered in the student's mind and be subject to recall when he himself essays the teacher's task. Thus his whole education sensitizes him to the learning process; it is not too much to say that the skillful teacher is one who can recall most clearly the successive steps of his own mastery and through these reconstruct in imagination the situation which the pupil is facing. The teacher who cannot do this is the teacher who is likely to leave out essential stages in instruction and then to charge up his failure against the stupidity of his pupils. It is just this power of recall and of self-analysis in fresh learning that explains the humility and sympathy of the learning teacher as contrasted with the mental snobbery of the teacher who does not insist that he himself from time to time attack strange and difficult material."

Recommendations. Including those which have just been set forth, the recommendations of the committee are as follows:

1. *That the normal schools of the state ascertain what students on entering are below the standard of the eighth grade in arithmetic and provide not less than a three-hour course for these persons for the first semester.*

2. *That for those of Group One who are not preparing specifically to become kindergarten teachers a three-hour course be provided in the second semester in The Teaching of Arithmetic.*

3. *That four hours instead of three hours be given to The Teaching of Arithmetic for Group Two, second semester.*

4. *That a three-hour course be given the Rural Group in the third semester embodying such phases of arithmetical work as are closely associated with rural life,—in the main, farm arithmetic.* There is no subject in the curriculum of the rural schools in which the teacher is tried out more severely and with such varied problems as she is in the subject of arithmetic. The meager training provided for her by the present course leaves her unprepared to meet situations with which she will be confronted.

5. *That members of the Junior High School group be given a general course in the subject-matter and teaching of arithmetic for three hours in the first semester.* The reason for this recommendation is, in the main, that many persons taking the Junior High School course will not be able to secure positions in Junior High Schools; consequently, they will find it necessary to teach in grades for which they may have had little preparation. While statistics are not available from the different normal schools, the committee is of the opinion that many who have already completed the Junior High School course have not found employment in Junior High Schools. The objection may be raised that students do not select their groups in time to do this; but the committee answers that all who expect to teach in Junior High Schools should be asked to make their selection on entering school.

6. *That three hours be allowed for free electives in each semester for all groups, except in the first semester for Groups One, Two and Three, and that such adjustments of the present course be made as will provide for this without increasing the number of periods of recitation per week.* In this connection, it is recommended *that in the third and fourth semesters of the Junior High School Group,* 3-3 *courses be provided as well as* 6-6 *courses.*

7. *That all persons who specialize in Junior High School mathematics be required to pursue a three-hour course in the*

sixth semester in the teaching of Junior High School mathematics.

8. That each normal school be free to offer the following courses as electives in mathematics, and any additional courses deemed desirable by any of the normal schools:

Intermediate Algebra	3 hours
Plane Trigonometry	3 hours
Plane Trigonometry and Surveying	6 hours
Solid Geometry	3 hours
Advanced Algebra	3 hours
Teaching of Junior High School Mathematics	3 hours

The committee's excuse for this lengthy report, if an excuse is necessary, is founded on the fact that apparently the subject of mathematics had no one at court to plead its cause when the present course of study was decided upon; in fact, it is barely evident that its case was given consideration at all. As has been suggested before in this paper, situations which the teacher must face must not be disregarded in making up our courses of study; it is unwise to send teachers out lacking in preparation for the teaching of a subject which the people yet regard as of vital importance and one which is frequently used as a gauge of the teacher's ability and by which she is classed as a success or failure according as she teaches it well or poorly.

West Chester, Penna., March 27, 1923.

R. F. Anderson, *Chairman,*
I. F. Seiverling,
J. W. F. Wilkinson,
J. Seth Grove,
J. A. Foberg,
Committee.

IMAGINATION IN MATHEMATICS

By PROFESSOR W. V. LOVITT
Colorado College, Colorado Springs, Colo.

A constructive imagination is one of the most important elements in our life of today. It is the man with an active, vivid imagination who is achieving the apparently impossible.

James Hill was endowed with a tremendous imagination. He had a vision of the development of the Northwest under the stimulating influence of a railroad. It was this vision which gave him the courage to attempt to build railroads into this region.

The Wright brothers and their sister, who financed their first experiments with the airplane, were possessed of large imagination.

The men who have developed wireless have been men of imagination, men who not only could see the immediate usefulness and possibility of talking across vast distances but who were possessed of a fore-knowledge, due to their imagination, of its many other possibilities.

The naturalist to whom the meadowlark says in its song, "Spring's come! Here we are! Here we are!" must be possessed of a charming imagination, an imagination which gives life to the trees and rocks and the wind.

The ancient astronomers saw strange animals and figures outlined in the sky. There was the Great Bear and the Little Bear. There were serpents and lions in the midnight sky. There were armed warriors with uplifted arms poised to strike the many-headed Hydra. There were kings and charioteers and queens. There was Cassiopiae seated in her chair and Bernice with her beautiful hair. Of a surety they were possessed of a wonderful imagination.

My little six-year-old daughter comes running to me and says: "Papa, do you see the cracks in my hand? I think that must be where God put us together, don't you." She has an imagination. To one with such an imagination the world must present many pleasing aspects forever hidden from us of lesser imagination.

Baron Munchausen must have possessed an imagination which ran riot. His imagination, I have no doubt, kept him constantly amused at the possibilities of the most common incidents.

The detective should have an active imagination to aid him in reconstructing the mental processes and activities of the criminal.

The real estate dealer, to be successful, must be able to see in advance the possibilities of a given locality. Seeing these possibilities he buys a plot of ground in this locality and then proceeds to endeavor to make others see his vision. This is not so difficult. We can all make an egg stand on end after Columbus has shown us how.

Our schools and colleges have many teachers of English and English literature. These teachers have many pupils. Do they write? Not in any great numbers. Why? They are lacking in imagination. A Texas cowboy with only a grammer school education but possessed of an imagination which would delight a Baron Munchausen can and does write stories which are an impossibility to a doctor of letters, lacking in imagination.

For writers, the imagination is the one thing without which is nothing. Stevenson's *Dr. Jekyl and Mr. Hyde* is a product of the imagination. Any ordinary correct English is sufficient to make this story last. The appeal is to the reader's imagination.

Jules Verne appeals to our imagination with his. Ninety per cent of the attractiveness of the Jules Verne stories is due to their stimulus to our imagination.

Poe was a master of the short story. He is excelled by many in the art of expression, but surpassed by none in power of the imagination. He is at his best in *The Pit and the Pendulum* and *The Cask of Amontillado.*

O'Brien's *The Diamond Lens* is a masterpiece both of expression and imagination. However, the essential appeal to the reader is through the imagination.

Stockton's *Negative Gravity* is a delightful story conceived by a riotous imagination. The reader is kept on the qui vive by the succession of astonishing events which occur to the inventor of negative gravity. It is the delightful absurdities which hold the attention and not the mode of expression.

One of the best baseball writers of any time was Joe Campbell, for many years with *The Washington Post.* Language failing him, he invented it. He wrote, "and Annie Russie made a Svengali pass in front of Charlie Reilly's lamps and he carved three nicks in the weather."

No doubt O'Brien received during his college days in his study of biology the elements from which his imagination constructed the story of *The Diamond Lens.* Stevenson had as a background for *Dr. Jekyl and Mr. Hyde,* readings in psychology, especially that which related to double personality.

However vivid the imagination, it must have material with which to build. An active life as a sea captain, a Texas cowboy, an engineer, or a mathematician are equally good in furnishing elements for the work of a creative imagination.

Now the question seems to be, how can mathematics stimulate the imagination?

Washington is 77° 3′ west of, and 12° 35′ farther south than Greenwich. Nevertheless the bearing of Washington from Greenwich is west 18° 33′ north. That is, in going from Greenwich to Washington by the shortest path on the surface of the earth, namely an arc of a great circle, one would start 18° 33′ north of west from Greenwich.

This result is not intuitive. The greater the difference between the facts and what our intuition tells us ought to be so, the greater is the appeal to the imagination.

The symbolism of mathematics is not the least of its appeals to the imagination. Mathematics has been facetiously defined as that subject in which you do not know what you are talking about and do not care whether what you are talking about is so or not. The symbol x of your algebraic manipulation can stand for anything which can be counted. You do not know, and what is more to the point you do not care to know, what things can be represented by the symbol x. You do know that if a thing is subject to representation by the symbol x and if x satisfies the first equation written down, then it must satisfy the other equations which you deduce from the first.

There is a strong appeal to the imagination when one learns for the first time that mathematics furnishes the means for measuring the distance to the sun and moon. We know their diameter and weight. We know the height of Mt. Everest, although as yet no man has set foot on its hitherto inaccessible summit.

The mathematician computes the orbits of the heavenly bodies. Long after the strongest telescopes lose track of the comets the

mathematician follows them, and can and does predict the time and place of their return to telescopic vision.

In 1845 Adams and Le Verrier independenly and by different methods, by a mathematical computation, indicated the existence and position of the hitherto undiscovered planet Neptune. On directing the telescopes to the part of the sky indicated the planet was found. What greater appeal to the imagination than this?

The so-called imaginary numbers stir the imagination. A kick from a mule you would not consider an imaginary thing especially if it struck you squarely between the shoulder blades. It is a force of definite magnitude and direction. As such it can be represented by a complex (imaginary) number.

Magic squares and cubes present a definite appeal. The presence of the word magic is sufficient evidence of this.

In geometry we sometimes use the term inventional geometry. Here the constructive imagination is called into play. The elements of geometry are united to form the various ornamental architectural designs. Witness the decorative windows, doorways, and arches. Combinations of straight lines and circles make up decorative borders and the patterns for oilcloths, linoleums, and parquet flooring, whether of wood or tile.

We all know that if ten men have a dollar each and give half of it away in no possible way can they still have a dollar each. But if we have a line consisting of an infinite number of men and they have a dollar each, it is possible to take away half of their money and still to arrange so that every man has a dollar. With finite numbers the whole cannot be put into a one-to-one correspondence with a part of itself but with the infinite a part can be put into a one-to-one correspondence with itself. Consideration of the infinite stimulates the imagination.

It is with some surprise and much speculation that we first learn that there is a geometry devoid of measurement, a geometry wherein relative position only is taken in account. We are again surprised to learn of a geometry consisting of a finite number of points and lines. A series of surprises awaits the student of mathematics. Parallel lines meet in an ideal point at infinity. There is such a point on every line. The totality of such points in a plane constitutes an ideal line at infinity.

The totality of such points in three space constitutes an ideal plane at infinity.

Non-euclidean geometry again stimulates the imagination. It is with something of a shock that a student first learns that it is possible to get along without the assumption that there exist lines which do not meet. The shock is greater when he learns that the angles of a triangle no longer add to 180^0 but to more or less, depending on your hypotheses.

Geometry of four dimensions or of more dimensions has a strong appeal. The first appeal seems to come from philosophical considerations. That the appeal is there is evidenced by the publication of such books as *Flatland by A Square*. This book is read by a great variety of people. Further evidence of the generality of the appeal to the popular imagination is the prize contest held several years ago by the *Scientific American* for the best popular essay on the subject.

No one subject has made a stronger appeal to the popular imagination than the *Theory of Relativity*. This is essentially a mathematical development. The average mind is incapable of understanding the development of the theory. However, some of the results can be put in simple terms understandable by all. For example: space is finite; our size changes as we move about; length in the direction of motion decreases with the speed; events cannot happen simultaneously. These statements are so at variance with our previous notions that they have aroused the popular imagination to a high degree of activity.

The mathematician draws mentally many construction lines in making a geometric proof. Many of these lines he rejects as they lead to no result. The more actively imaginative individual will draw and reject several construction lines and arrive at a useful construction line long before his less imaginative brother.

I can see no hope for those who aspire to be an architect or painter if they cannot master solid geometry. My experience is that the main difficulty with solid geometry is not the logic involved but the inability to visualize the three dimensional figure, a drawing of which is made on a plane surface. Now the engineer and the artist must of necessity make a two-space drawing of a three-space figure. A design for a building, or a bridge must first be created in the author's mind by his creative imagination.

Try to count the number of vertices, edges, and faces when a small piece is cut by a plane from each corner of a cube. Ask your friends this same question and you will find that many of them are unable to give the correct result. There are 24 vertices, 36 edges and 14 faces.

Enough instances have been cited to show that mathematics does excite the active imagination. Indeed the cases cited above contain the elements for innumerable novels. Here are the elements for stranger tales of mystery and imagination than were ever penned by an Edgar Allen Poe or conceived by the riotous imagination of a Baron Munchausen.

A constructive imagination is the one thing which an individual must possess if he is to rise above the level of those who imitate, follow the leader, or work by rule of thumb. A constructive imagination one must possess if he is to lead. Otherwise he must be led.

FROM THE SHELVES OF DR. DAVID EUGENE SMITH'S UNIQUE MATHEMATICAL HISTORICAL LIBRARY

By SOPHIA R. REFIOR[1]
Scott High School, Toledo, Ohio

The students in the seminary in the History of Mathematics under Dr. David Eugene Smith have been privileged at various times to visit the unique private library of their instructor and to use such manuscripts as were necessary in their field of research. The inspiration derived from examining these priceless objects has stimulated one of the pupils to write the following account. This theme is restricted to the selected portion of the library which the class viewed and which is only a small part of the entire collection.

This library of historical, mathematical material is one of the most interesting of its kind in the world, not only because of its size, but because of the very rare volumes upon its shelves. Books of all periods of history and of all countries are found there. Early historic tablets, original manuscripts, autographed letters, presentation copies and first editions as well as rare translations are included in this excellent library.

Dr. Smith has personally collected these rarities during his numerous travels in Europe, Asia, Africa and South America and has spent much time and money as well as care in selecting them. He has received them from friends or bought them in bookshops, but more often he has ingeniously gathered them from remote and obscure places, with the view in mind of preserving them for his students. The collection is more than a collection,—it is an expression of the interest and personality of Dr. David Eugene Smith.

This Library offers original source material on various subjects that cannot be found elsewhere. Hence, it vitalizes the history of mathematics for all students, but it is of priceless value to research workers.

It is difficult to choose a few books for a description from so vast a store of riches, for any selection which might be mentioned in this brief article would necessarily omit hundreds of equal merit. However, an attempt will be made to describe a few of the rarest pieces.

[1] Now at Teachers College.

From the standpoint of antiquity, the clay tablets are of unusual interest, dating from about 2350 B. C. The smallest are about one and one-half inches square and one-half inch thick while the largest are three inches square and one inch thick. On either side are the letters and numerals that were impressed with a sharp stick while the clay was soft. They are tax lists, receipts and tablets of school children.

The history of mathematics becomes a very live subject when one can personally handle objects of so many types. There are various examples of English tally sticks, the rarest specimens being some that had been found in the Chapel of the Pyx, Westminster, London. They had lain there for 600 years, having escaped the order of 1834 that all English tallies be burned. These tallies bore the name of William de Costello, who was sheriff of Londdon in 1296.

The idea of magic and mystery in mathematics as it appeared to some people is illustrated by the Thibetan *Wheel of Life,* which is printed on rice paper, oblong in shape, twenty-two inches by thirteen inches. This sheet of block printing which was done at Lhassa, represents a magic square, surrounded by the Pakua, and then the signs of the Zodiac. One of the Chinese pieces of magic is a rectangle of fine white silk cloth two yards long and eighteen inches wide. It contains the trigrams, a copy of the *I-king* and of the oldest magic square known, one in which numbers are represented by dots, the masculine or odd numbers being white, while the feminine or even numbers are black.

Dr. Smith's library represents the great mathematicians as very human because it contains many books that were presentation and association copies of which three will here be mentioned.

L'Introduction au Cours de Mécanique Appliquée aux Arts, by Ch. Dupin, is autographed by Dupin and presented to Archibald Constable (1774-1827). Constable was at one time owner of the Encyclopedia Britannica, and it was his failure that bankrupted Sir Walter Scott.

L'Histoire des Science Mathématiques en Italie, by Guillaume Libri, was printed in 1835. After the first volume was printed, the printing establishment burned and all but eight of the copies were destroyed. The author set to work to prepare the copy for another printing, so he embodied changes in the notes and minor

changes in the text. Years afterward, Libri gave this copy with corrected notes to his friend, Professor Jacoli, then living in Venice, and from him Dr. Smith received it in 1904. Very few copies of this edition can possibly be extant, and probably none with Libri's notes.

The Analyst, written by Dean Berkeley, is shown in a copy autographed by him and given to Sir Thomas Hammer, Speaker of the House of Commons.

Dr. Smith has also about 2,000 original letters of many of the greatest mathematicians of the last 200 years, including Delambre's letter which led to the release from prison of James Smithson, founder of the Smithsonian Institute, Descartes' appreciation of Huygen and various epistles concerning the French Revolution, the development of the metric system, and other themes of vital interest. Such names as Leibniz, the Cassini, Lewis Carroll, Montucla, the De Morgans, the Bernoullis, Bossut, Delambre, Legendre, Laplace, De Moivre and Sylvester are of frequent occurrence. In the list of rare letters and manuscripts, two documents signed by Sir Isaac Newton may also be mentioned.

Invaluable to the student as are all the ancient manuscripts of Dr. Smith's library, only a few can be mentioned in this brief article. Of Bhāskara, alone, the most prominent writer of Hindu mathematics from 1000 to 1500, this library contains eighteen manuscripts. The oldest one is a copy of the *Lilavati,* the author's most celebrated work. It was written on long, narrow strips of palm leaves, in about 1400, before paper became a common medium. To secure greater durability, the leaves were placed between wooden slabs and were tied by strings running through the books. At present they are tied by linen tapes, weighted by spherical coins—the Siamese Ticuls. The manuscript is still in perfect condition as well as the other manuscripts of Bhāskara, all of which extend chronologically from the year 1400 until late in the nineteenth century. They are generally written in Sanskrit on loose leaves of paper eight inches by four inches, held together merely by a cord; although three are in Persian and are bound in the ordinary book form.

This library contains the complete Sūrya Siddhanta with commentaries written on loose palm leaves during the fifteenth century. It also contains the proof sheets of Mahāvīra's note-

worthy contribution as they were going through the press; and a Madras manuscript of the complete works of Aryabhata. Among the greatest rarities are a manuscript of the algebra of Mohammed Mūsa and one of the algebra of Omar Khayyam.

This library contains many Persian, Arabic, Hindu and other oriental manuscripts on arithmetic and algebra, and many others which are of such rarity that they deserve mention although they do not relate strictly to mathematical subjects. A few of these are as follows: A Persian manuscript of about 1550 which contains a commentary on the astronomical works of Prince Ulugh Beg; some vellum-bound books of four-bar music; a Spanish genealogical piece on parchment; numerous Coptic pieces; a rare Buddhistic manuscript entitled Darjechepa with Thibetan numerals in the pagination; one with the Buddhistic scripture lacquered in sepia on gilded tin plates; another of the same subject printed on a fabric made out of the robes of deceased priests; and also the Book of Ruth on Morocco leather from Tangiers.

For intrinsic beauty as well as for its historical and literary merit, a copy of the *Rubaiyat* of Omar Khayyam, written in the sixteenth century is particularly noteworthy. The cover is of soft leather, gilded and the leaves are of Arab paper of a very durable quality and similar to fine parchment. The body of the text is in black while the capitals are gold, making it literally as well as figuratively a jeweled book.

Of books of our modern style of binding, some of the old ones are very large—the size of a family Bible. One of these is a copy of Boethius' first and second folio editions, parts of which were printed in 1491 and parts in 1499. It is bound in vellum and is still in perfect condition. Pacioli's *Summa*, which appeared in Italy about the time America was discovered, is in folio, each leaf having but one number. Vellum, which was so frequently used in the fifteenth century, made a durable binding but made the books remain partially open. For this reason, clasps were used to fasten the books. This custom has been preserved until today in our family albums and Bibles.

The four massive volumes of Clavius are bound in pressed pigskin and closed by such clasps. The title page has many pictures of the astrolabe, quadrant and other mathematical in-

struments, as well as one of the author. Dr. Smith has three editions of Cardan's *Ars Magna,* including the first one, which appeared in Nürnberg in 1545.

The books, however, which interest us the most are the various editions of Euclid. Dr. Smiths' library contains about seventy-five different editions of Euclid's geometry besides two arabic manuscripts of the thirteenth and fourteenth century and the Matteo Ricci translation into Chinese, this copy dating from the seventeenth century.

Dr. Smith has also a copy of the first edition of the first English translation of Euclid, made by H. Billingsley, a citizen of London, and edited by John Dee in 1570. In this Euclid bound in old calf, the theorems are written in narrative forms with the diagrams included in the description. There is no formal separation between statement and proof or between fact and reason. The initials are beautiful wood engravings. Like all early editions, the book contains theorems and construction problems but no original problems.

The most curious features of the collection are the Persian and Chinese editions. Tabit ibn Qorra revised a Persian translation of Euclid about the year 890. Dr. Smith's copy bears the date 751 A. H. which means 1335 A. D. in the Christian calendar.

The editions of Euclid in Dr. Smith's library include texts in Latin, Italian, Greek, French, German, English, Persian, Chinese and numerous other languages. All books contain practically the same subject matter but differ in the arrangement of material, in the preface and in the notes.

MEMORY AND MARKS IN MATHEMATICS[1]

By ETHEL LUCCOCK
Northwestern High School, Detroit, Mich.

Last November I read an article in *School Science and Mathematics* on "The Relation Between Thinking and Memorization in Mathematics." It was written by Mr. E. W. Atkins of the Carnegie Institute of Technology. It raised the question in my mind: Do I make my work depend too largely on memory? Should I change this? Should I change the plan of work to give more definite training to the pupil's memory?

We discussed the question in an algebra class. It was a senior class, an elective course, and in membership represented the different elements of the school. It was interesting to note their reaction. Those who make the highest grades had nothing to say. One boy, of pure French descent, and a scientist to his finger tips—he had a radio long before the newspapers in Detroit had them—rose with a protest against examinations that were entirely memory tests, as in history where definite dates were asked for. The rest of the class spoke out that all the history tests were not like that. They seemed agreed that a test that depended on rote memory was not the best form of an examination. Another boy, a quick, clever lad who puts in his afternoons selling window shades, declared that it was the ability you developed that counted. The budding engineer who sat near, immediately wanted to know what good your ability did you when you could not remember things accurately.

They represented the weakness in our teaching of mathematics. We have looked upon mathematics as pre-eminently a subject that trains the pupil to think. But, as the writer referred to observed in this article, "In mathematics it is exceedingly likely that teachers have gone so far in attempting to get their students to think that they have overlooked the function that memory performs."

A teacher of psychology gave as her opinion on the relation of marks to memory that good marks were probably not dependent on good memory. The child that has a good memory has prob-

[1] Read before the National Council of Teachers of Mathematics, Feb. 28, 1924.

ably a high intelligence also. The correlation between intelligence and ability in mathematics is so high that mathematics questions are used to test intelligence. But poor marks may seem to be due to poor memory. The pupil is kept from understanding the new fact by his inability to recall the old accurately.

How often in teaching geometry do we come to this equation in proving the area of a triangle in terms of its sides:

$$\frac{AD}{b} = \frac{c}{2R} \text{ and go on to } AD = \frac{bc}{2R},$$

to meet from half a dozen pupils, "I don't understand." Or again in the area of a triangle in terms of its sides if we use the expression

$$c^2 - \frac{(b^2 + c^2 - a^2)^2}{4b^2}, \text{ and follow it with } \frac{4b^2c^2 - (b^2 + c^2 - a^2)^2}{4b^2},$$

the same pupils are all at sea. I was working with a boy who was very weak in algebra. In a fractional equation he came upon the binomial $b^3 - 27$. "There," he said, "I can't do that." I suggested the one factor $(b - 3)$ and he gave as the other $(b^2 + 6b + 9)$. When I corrected the 6 he recalled that he had been corrected on the same thing before. I think we will find Thorndike's *Psychology of Algebra* full of suggestions as to specific things in which we need more drill as for instance on page 442 where there are some fractions showing the type of problem on which to drill to prevent the pupil from cancelling terms instead of factors. We need wise practice to produce certain abilities, and to give more attention to the fundamental bonds.

In trying to determine the relation between memory and marks for my classes, I planned my two mid-semester tests in a contrasting way. The first was a series of twenty or twenty-five questions that depended on memory of fact or rule, both in geometry and algebra. The second day the questions in geometry were propositions and exercises to be proved and in algebra, problems to be solved. I plotted the results to see if there seemed to be any connection between poor memory and poor work in the formal proof or problem solving. One geometry class was a specially selected group. Many of them had come through special classes. The other geometry class had only two of the highest ability, that is, those who ranked 1 in our system of marking. In the special, or "fast" group as we call

them, the memory test seemed very little higher than the other. In the weaker class the difference between the two seemed greater. In the individual percentages, three times in the weaker class the second test was higher than the memory test and four times equal to it. In the stronger class six times the second test was higher than the first and four times equal to it. In reading their papers, I found them less dependent on memory. Their proofs were often original, or if they used the facts used in the book, the arrangement was their own. The graph of the second test did not as closely follow that of the memory test.

In the algebra class the second test seemed to follow the first in its downward tendencies, but while I had no hundred per centers in the memory test, I had eight in the one on problem solving.

In the weaker geometry class I worked very hard helping them to trace the associations of their ideas. When a fact was incorrectly used we traced the association of the idea and tried to show that the association was incorrectly made. We had very little written work, but much informal discussion with oral proof. The final examination showed a very different class. The mid-semester examination gave a median of 52.5%, and the final, a similar examination of twice as many questions and double time, gave a median of 84. In the special group the corresponding medians were 75.1% and 94%.

I will cite one of the cases individually studied. The type occurs in every class. Marion M. says she has a very poor memory. She could not memorize poetry. She could not get along in history at all, yet she can do fairly well in Latin. There she seems to recall the meaning of words through the meaning of sentences. In geometry she did not seem to have the usual word associations that the student forms. "Equally distant from" suggested nothing at all. Her associations from the figures or diagrams were all right. She could not complete the statement "When two straight lines intersect," but when she turned the sentence into a diagram she immediately said, "the opposite angles are equal." One time she could not understand the proof in the book, so she worked out an original proof. This she remembered easily. She gets a great deal out of oral recitation but little out of written propositions. On tests she fails utterly because she cannot start things.

I suggested to her that she should not try to memorize statements, but put them into the form of question and answer, as, equal central angles—do what? Intercept equal arcs. Also that she would do well to try all her proofs as originals, going to the book for help only. Then if she would turn the statements into drawings, as far as possible, she could remember from the drawings. Her final examination showed improvement, 60 per cent, as against 25 per cent in the mid-semester, and I am hoping that in the next course with so many more applications, she will do vastly better.

Homer S. presented a different problem in algebra. He was a colored boy who had had his early training in the South. He was working eight hours a day at Ford's outside of a full school program, from four till midnight. At the beginning of the term his tests were sometimes absolutely worth nothing. But still I was sure he had ability. The second month I called him in and told him he must get over the feeling that he was handicapped by his poor preparatory work, that he could do it all right. I threw his tests in the waste basket and gave him 2. With the element of fear removed he began to do better. He still often made mistakes due possibly to fatigue. But he would take his paper and analyze his mistakes and come in the next day and write another test usually scoring about 80.

From this experience I have tried to get the algebra students to see that each one had a standard of his own, that he should make himself measure up to. After mid-semesters I had the whole class make graphic records of their tests. When they had made up poor tests it gave a fairly even standard.

Another way in which we can help a class in algebra is to be very definite in the requirements of the course. Half way through the course I gave to the class an outline of the things they would be held responsible for. I made my final examination from this outline.

We need to change our method more with respect to the ability of the pupil. Psychologists tell us that we learn by the vividness of the impression and by repetition with satisfactory results. The best student usually gets a sufficiently strong first impression so he remembers readily, recognizes the same situation when he meets it again, and handles it easily. The weaker student has not been like his stronger brother, eager to learn, so

he does not get the same vivid first impression. He learns largely through repetition. Now it is here that Thorndike has given a striking service in his "Psychology of Algebra" in showing how drill can be most effectively used by repeating at stated intervals. The chapter on "The Distribution of Practice" with its illuminating diagrams should be carefully studied by every teacher of algebra. One should separate out the new elements of each course and make those thoroughly mastered by properly spaced drill.

I tried to make a beginning of this in an algebra class this year. I had two fourth semester algebra classes. They make a complete review of their earlier work in algebra and do some advanced work. A part of this new work consists of the progressions and the binomial theorem. In one class I repeated the three new types of problems in some way once each week. In the other I reviewed them only at the end of the term. In their final examination in the latter class, 79 trials of the new problems were made, 26 were incorrectly solved and 53 correctly. In the first group 82 problems were tried, 17 were incorrect and 65 correct, a percentage of 20.7 incorrect, as against 32.9 per cent incorrect in the other group. Of the two classes there were more of good ability in algebra (as shown by the Thurston test and various class tests) in the group that failed in the larger number of problems.

The weaker pupil learns, then, through repetition and when this is properly distributed it does not harm the bright pupil. The good student likes being tested out and making his hundred per cent.

He may be harmed, on the other hand, by improperly distributed drill. I used to get pupils from a teacher who massed all her work on a given topic at one time. Her classes stayed on factoring four or five weeks and then knew nothing about it. They progressed straight through page after page of problems like an army advancing. I received from her algebra class two girls whom I had previously had in geometry. I had found them to be students of the highest ability. They could do execellent work on originals. When I got them from this algebra class it seemed impossible for them to attain the same standard they had achieved in geometry. There were positive limitations to what they could do, old misunderstandings that should not have

occurred to them at all. I believe they were really harmed as students by drill improperly handled. They were "stale."

A good student can also be spoiled by over-speeding him. We have grouped our best students in special classes as far as possible. Some were formed in the seventh and eighth grades and some in the high school. The period of study for algebra was shortened in these groups, some of the high school classes doing two semesters' work in one. For two years I have used the Thurstone Vocational Guidance test in my fourth semester algebra class. Students from these special groups have never scored as high in this test as those who have had the full three half years' course in algebra. They seem to lack sureness. I have never ranked one from these groups as a 1 in this last course in algebra. And yet they were put in those groups because of special ability. They seem to have been overcrowded and hence suffer for it.

I had in connection with this same test a striking illustration of the need of proper drill. One of the questions is a fractional problem that gives a result 4/0. Our text does not treat that at all. The question had come up at the beginning of the term when, as always happens in a certain type of problem, they did not know whether the answer was 1 or 0. I asked what 7/0 would be. I showed how decreasing the value of the denominator made the value of the fraction increase, and if the denominator should actually reach zero the fraction should have an infinitely great value. It came up again toward the end of the course in plotting the graph of the equation $xy = 36$. We let $y = \frac{36}{x}$ and when x was zero showed how the hyperbola approached infinity. There is no doubt that a clear impression was made, yet when that problem came up in the test only two days after the last reference, only four out of forty remembered to write the sign for infinity for the value of 4/0, and in another group, two out of thirty-four, while two others wrote 4/0, recognizing it differed from 0. It was a perfectly new idea and they had made no use of it. It simply passed out of mind. A despairing mother once protested against her little daughter's heedlessness, saying things just passed in one ear and out of the other. "Of course," said the child, "that is what I have

two for." Possibly it is necessary for the child's protection, but information travels in a very straight path unless it is made to form some lasting associations.

Many of these things seem obvious, but go through the average high school and you will find the prevalent method is to go straight through the course and spend some time reviewing at the end, and feel that one's duty to the class is discharged. Some teachers make a custom of a weekly test. This is a great help to the teacher in determining a pupil's industry and his proper grade. I am afraid most of us do not use the weekly test intelligently. It would be useful to show what points of the week's work needed further stressing rather than to determine marks. A better plan I think for geometry might be to have some one thing from the week's work written and cover the rest through oral work. One finds many incorrect ideas brought out in an informal peroid of review that a test does not show at all.

We have criticized the class room teacher. Yet there are many reasons why we do not improve our technique of teaching as we might. One cause, of course, is the size of the classes. The school boards in all the large cities are hard pressed for funds and the classes have increaed in size. The average teacher carries about one hundred and fifty to two hundred pupils a day. Yet I do not think the pressure of work alone would discourage the teacher. It is the lack of continuity in our work that hinders our improvement, and the fact that we do not see the result of our efforts. We work with individual pupils through one half year and then a new semester's program sweeps them on to another teacher. Or we work with a class and try out different ways of handling the course and the group is broken up and scattered through a half dozen classes. We were trying an experiment to compare the results of a year's work in geometry done by a single period class with those of a double period class where the preparation was all done under the class room teacher's supervision. We found it impossible to keep the identical groups together. Even the teacher had to be changed. Our work would be more constructive if it could be more continuous. When you are not able to check the results of your work, your reaction is—Oh, well, what's the use. The first few years as a teacher I kept over my desk a copy of a cartoon by Lewis Carroll, the author of *Alice in Wonderland,*

who was, by the way, an eminent mathematician. The cartoon showed a large man, greatly agitated, wildly pulling out huge handfuls of his bushy hair. Underneath was the caption, "Alas, What Boots It." I have chuckled over that many a time and lost the feeling of irritation the teacher feels when the organization crowds her in.

I feel strongly the need for a more comprehensive type of a final examination over each semesters' work in geometry. I dislike the element of chance that enters into the customary four propositions and four exercises kind of examination. I feel with Helen Keller, "It is wonderful how much you know that they never think to ask you for in an examination." Do you remember the young hero in *Roughhewn?* He had spent a long time cramming for his entrance examinations to Columbia. After they were over he went up in the country and was content to spend whole days just watching the pigs. It seemed supremely satisfying to a mind tired out from poorly balanced study, from the futility of amassing a lot of facts for the few that he found required of him.

As I study the pupils, and I have about one hnudred and sixty each day, I become convinced that the pupil of higher ability seems less dependent on his memory than his weaker classmate. If he forgets his formulas or laws he will work them out again. The weaker student becomes confronted with something about which he feels, "There, I don't know that," and his brain ceases to work. I started out with a boy last September who did not seem to remember anything in algebra. I was about to have him drop the course, believing with Thorndike that operating with anything to the extent of being right only once out of four times reduces the value of training in algebra to the zero point. But in the familiar topic of numerical square root he seemed to find himself. He made one hundred on a test. It was just as if he had been struggling in the water unable to swim and suddenly found his feet on something solid. He used to drop in when I had a vacant period and work, picking out things he could not do. He made satisfactory progress, and a final examination score of 79.

The pupil with the weak algebraic bonds is confused in mind and cannot think. "The present psychology finds the mind is

ruled by habit throughout. It defines reasoning as the organization and cooperation of habits rather than as a special activity above their level.'"[1] So our weak pupil is more apt to reach correct conclusions when his recall is accurate. It is his approach to thought.

[1] Thorndike—Psychology of Algebra.

MATHEMATICS CLUBS

(Prepared by Misses Russell, Duncan, Gulden, Symmes and Derby, Graduate Students in Teachers' College.)

Purpose of the Club. The purpose of the Mathematics Clubs should be to round out and supplement the mathematics courses in the high school. The length of the ordinary recitation period is not more than three-quarters of an hour and the time is usually spent in drilling on just those principles which the Board of Regents and College Entrance Board have decided are essential. Many students will find that they particularly like and enjoy mathematics and these students should certainly be given an opportunity to try out and make use of their special powers in this subject. We can afford them this opportunity by means of the Mathematics Clubs. In fact we think the club will do the following for the students:

1. Specially good students may form a group to continue with the study of branches of the subject not taught during the regular session. Many students have done so in their leisure time, and have taken the required Regent's examinations and made extra credit towards a college degree.

2. The club is going to help the less fortunate members who are experiencing difficulties with their work in mathematics. A "help class" will be formed—the proficient students acting as student teachers to help the slower ones. Special credit or special privileges will be granted those who administer such aid —and they will enjoy the work immensely.

3. The club is going to afford opportunity to the students to see mathematics from different angles and points of view than are usually touched on in school:

a. They will be interested to learn something of the history of the subject—its beginnings and development in different countries; ancient methods of reckoning and counting and the survival of these methods which we have at the present day.

b. They will be interested in the practical applications of mathematics in every day life; in learning what a training in this subject will fit them for; what the people who have specialized in this subject are doing.

c. They will be given a chance to see the "fun" of mathematics—to learn its games, puzzles and fallacies.

d. Last but not least we are going to have some interesting mathematical socials together—with as many as possible to be held out of doors. In this connection we will become acquainted with and learn to use mathematical instruments.

Organizing the Club. In order to make our Mathematics Club a success we must see that it gets a good start. We want everyone in the school to know about it and be interested in it for we are going to open membership to every single child in the school.

The best time for bringing the club to the attention of the students would be at the weekly student assembly. We shall ask to have one of these assemblies given over to the Mathematics Club and we shall promise to entertain for that day—relieving the English and music departments who usually shoulder the burden of this work. We shall furnish the material and enlist the aid of these departments to help us "put it over." There are many things we might do, as for instance—stage a little play like "The Adventures of X"—(enacted very successfully by the students in California)—or we might give a minstrel show with Mr. X and Mr. Y and Mr. Z in place of the usual "Mr. Jones, etc." The minstrels will entertain with mathematical jokes, conundrums, fallacies, games, songs, parodies. We shall ask the audience to guess answers to some of these. We might entertain with an angle dance—or have a mock school room scene and teach the audience some surprising facts about mathematics. We might have an interesting talk on "Flatland, the Two-Dimensional World." (Many suggestions will be found in the bibliography.)

At the close of the play a student will announce that we are going to form a mathematics club and will state (as above) just how we hope the club is going to interest and be of benefit to each student.

One of the most difficult problems will be to find a convenient time of meeting (when the lunch period lasts one hour, half of this time might be devoted to the club). This is not often the case, however, so that meetings will have to be arranged to suit the programs of the school. Wherever possible club meetings should be held during free school hours rather than after school. Attractive out-of-door programs arranged for Saturday mornings will bring many out. We would suggest that the club

be divided into groups according to varying interests and the individual groups arrange convenient meeting times.

At the first business meeting of the club a name should be decided on—Agat or Tagat (Arithmetic, Geometry, Algebra and Trigonometry) X, Y, Z, Triangle, Parabola, etc.—and a committee appointed to design some kind of inexpensive pin.

A President (Directrix), Vice President (Focus), and other officers should be elected. If the club be divided into groups a chairman might be appointed for each group or committee, as:

Entertainment Committee—To take charge of plays, games, songs, recreations.

Library Committee—(If there is no mathematics library the club might unite its efforts in getting one together.) This committee should see that the library contains as many as possible of the best and most recent books in mathematics.

History Committee—The members of this group are to give interesting talks on the history of the subject.

Vocational Committee—To investigate the practical side of mathematics—the relation of mathematics to the various vocations—inviting speakers to address the club along these lines.

Problem Chapter—Those especially interested in solving any difficult problems proposed by the members.

"Student Help" Committee—Qualified students who will volunteer some of their time to act as student teachers (sometimes actually taking the place of a teacher who is absent for a day).

Contest Committee—Students who will have charge of the mathematical contests—"open to all students"—to be held every term in algebra and geometry. There will also be a committee of teachers to select students to take part in *interscholastic* contests.

Prize Committee—To select suitable prizes for contests and to see to it that an honor roll of mathematics students be published in the school paper.

Lastly, all the mathematics teachers must be interested in the club and must keep the students informed of its doings and encourage them to join.

(*To be continued*)

DISCUSSION

An Example of Geometry Teaching by the Laboratory Method.—The problem of teaching the five books of plane geometry to a class in one year is looked upon as being a task requiring careful consideration and conscientious work by the teacher. Recently it was the fortune of the writer to have a section in geometry which from the very beginning presented a problem not readily solved. The response of the class was in no manner what it should have been. No progress was being made either by the teacher in getting the work across or by the class in learning the subject.

As the class was one of normal intelligence a particular study was made in order to remedy the situation. It was observed among other things that the class produced good work in the laboratory and shop. It was this characteristic which gave an idea to the teacher the development of which proved to be a solution of the problem.

The fact that practically any two boys in a class of 30 or 35 differ from each other in ability is so evident that comment is unnecessary. As the class was functioning well in its laboratory and trade subjects an equal division was made in the class. In one class were placed the brighter boys—in the other were placed the slower boys. While a division is made also in the laboratory and shops, no effort was made to make the same division here. The equal division was made arbitrarily for convenience and the grouping of any particular boy was made on the basis of his work in geometry during the first four weeks of the school term. Thereafter all blackboard work, whether formal demonstration, work in originals or constructions, was assigned to a pair of boys—one from each of the two groups. The work then was done by the two boys in mutual consultation much in the same manner as they would perform a job in the shops or laboratory together.

The initiative was taken invariably by the brighter lad. Hence the slower boy was usually required to give the oral demonstration, proof or discussion which followed. From the very beginning the interest of the boys in this work was quickened. Progress was made at the very start which continued throughout the

term. The effectiveness of the method may be judged by the fact that not one case of complete failure occurred, notwithstanding the discouraging outlook at the start when an unusual number of lads were headed for failure.

No boy had a permanent partner—they were changed daily and at random. By following this method the brighter lad had his opportunity while the weaker lad is strengthened. It is this slower boy who specially merits our consideration.

Only one disadvantage has arisen so far whenever using this method. With a group of 20 boys at the blackboard in consultation one with another at the same time, the classroom may become for a moment as noisy as a shop or laboratory. But this is merely the outcome of enthusiasm of the boys and no difficulty will be experienced in getting them to cooperate in making the noise a minimum. VAN ZANDT SHIPPY.

Central High School, Philadelphia, Pa.

The Reflective Process in Geometry.—In that most interesting book, *An Introduction to Reflective Thinking,*[1] five steps in reflective thinking are stated and illustrated. These five steps are:

1. *The occasion of reflection.* "A student thinks when he applies his knowledge to the solution of an original problem in geometry."

2. *The definition of the difficulty.* "Before any progress can be made in reaching an answer, the conditions of the question must be clarified."

3. *The rise of suggestions.* An item mentioned here is that "penetrating suggestions come only when the thinker is thorouhgly familiar with the background into which the problem fits and has had wide experience with similar difficulties."

4. *The mental elaboration of suggestion.* "When the suggestions have arisen, they must be tested by reference to foundations and consequences."

5. *Evidence in fact and conclusion.* "When the suggestion is accepted, after this imaginative grilling, as apparently true, the careful thinker seeks confirming evidence."

Now if all this is true, how close an application of these five steps can be made to the discovery of a geometric proof? Sup-

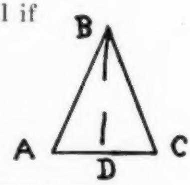
[1]Houghton Mifflin Co.

pose, for example, we have the following theorem to investigate.

1. *The occasion of reflection.* If two sides of a triangle are equal, the opposite angles are equal.

2. *The definition of the difficulty.* Given: AB equals BC. To prove: Angle B equals angle C.

3. *The rise of suggestions.* Angles are equal if



a. They are straight angles.
b. They are right angles.
c. They are supplements of the same angle.
d. They are complements of the same angle.
e. They are vertical angles.
f. They are alternate-interior angles of parallel lines.
g. They are corresponding angles of parallel lines.
h. They are corresponding angles of congruent triangles.

4. *The mental elaboration of suggestions.*

Which method will use equal lines? Only *h*. But there are not two triangles, Draw BD bisecting angle B. We now apply the third and fourth steps again to discover congruent triangles. Triangles are congruent if:

(3) (Two sides and the included angle, etc.)
(Two angles and the included side, etc.)

(4) (Can we get either of these requirements? The first.)

5. *Evidence in fact and conclusion.* The synthetic proof.

The third and fourth steps may be called the analysis of proof. In this analysis, the frist question is usually what methods are known for proving the desired conclusion? The answer is in the form of a summary of methods. The thinking is concentrated upon the class of propositions determined by the thing to be proved. Second, selective judgment is used. These propositions, one at a time, are examined and compared as to their fitness to be the method for the proof. The probable fitness can often be told by a similarity between the condition of the proposition under examination and the original proposition. The final selection of the method, however, is determined when it is found that all the requirements of its condition can be met. There may be repeated use of the two steps: (1) summary of methods; (2) trial of one of the summary. Each use may change the thinking to a different class of proposition. For example we may wish to prove two lines equal. This calls for a summary for

equal lines; then a trial of one of this summary. Suppose the proposition selected for trial requires congruent triangles; then, the summary for congruent triangles. One of this summary may require equal lines and equal angles. Unless some of these are given, two summaries will be necessary; and so on until a condition is reached that is found to be true.

Schultze says, "Students in secondary schools should be made to discover demonstrations by analysis but after this has been accomplished, the proof may be represented synthetically." And Thorndike, "Good teaching by deductive methods depends upon a clear statement of the goal aimed at, independent search by pupils for the proper class under which to think of the fact in question, criticism by them and by the teacher of the different classes suggested, and appreciation of the reason why the right one is the right one." Also Foster, "When properly taught, geometrical reasoning begins farther back than the demonstration of the theorem, in an analysis of the conditions involved; then by a synthetic procedure, the demonstration itself is constructed."

Here is the most important part of the class exercise both for the teacher and for the pupil: the development of new ideas, the exploring of new fields. Great care should be taken that the pupil always selects a method for his activities in forming the proof of a new theorem. There is little educational value in reading through a completed proof without knowing why the different steps should be taken before they are taken. Many pupils fail because they never understand that there is a very definite method guiding the work. The method of analysis discovers this method. Analysis for a direct proof consists in the repeated use of the question "The desired conclusion may be a consequence of what conditions?" until a condition is reached that is known to be true.

Of course a text book with proofs is not the best for this plan. The requirements are a syllabus of propositions and important questions arranged in groups with summaries, and an active teacher.

ROBERT R. GOFF.

New Britain, Conn.

UNIFIED MATHEMATICS IN SECONDARY SCHOOLS

By GERTRUDE JONES
Lincoln High School, Lincoln, Neb.

The reform movement in mathematics which had its beginning in England, France and Germany has been felt in this country for more than twenty years, and the end is not yet. In 1901 Professor J. Perry delivered his now famous address on the teaching of mathematics before a congress of mathematicians in Glasgow. Professor Perry was then in charge of certain apprenticeship schools in London. He felt that the mathematics which the students in these schools had studied did not function in their later work. Consequently there must be something wrong with the aim, content, and method of the traditional instruction in mathematics. The movement in this country was first started among college men by Professor E. H. Moore of the University of Chicago. In 1903 in order to spread the doctrine among classroom teachers, associations of mathematics teachers were formed in various sections of the United States and mathematical magazines were established.

The two last decades have seen a change in the aim of mathematical instruction as well as many improvements in the content of courses and in the methods of presentation. These changes have been the resultant of several forces, among which are new types of text books, the efforts of progressive teachers individually and in groups in experimental schools, and the investigations and reports of national and international committees. The most notable product of all these forces is the report on the Reorganization of Mathematics in Secondary Education issued by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America. This report is, no doubt, a compromise, and it was a disappointment to some, but we may say that, for the first time in the history of mathematics, we now have a national basis upon which to work. No one can foretell the extent of the future improvement in the teaching of mathematics which will come as a direct outgrowth of this report.

One of the constructive remedial measures which has been offered as a possible solution to the problem of improving instruction in mathematics is unified mathematics—or general, or

composite, or combined mathematics, as it has been variously called. Of unified mathematics the national report says (pages 12 and 13): "In recent years there has developed among many progressive teachers a very significant movement away from the older rigid division into 'subjects' such as arithmetic, algebra and geometry, each of which shall be completed before another is begun, and toward a rational breaking down of the barriers separating these subjects in the interest of an organization of subject matter that will offer a psychologically and pedagogically more effective approach to the study of mathematics. There has thus developed the movement toward what are variously called 'composite,' 'correlated,' 'unified,' or 'general' courses. The advocates of this new method of organization base their claims on the obvious and important interrelations between arithmetic, algebra and geometry (mainly intuitive), which the student must grasp before he can gain any real insight into mathematical methods and which are inevitably obscured by a strict adherence to the conception of separate 'subjects.' The movement has gained considerable new impetus by the growth of the junior high school, and there can be little question that the results already achieved by those who are experimenting with the new methods of organization warrant the abandonment of the extreme 'water-tight compartment' method of presentation."

Since there seems to be a confusion of opinion as to the meaning of the term "unified mathematics" it may be well at this point to define the term as it is used in this paper. By unified mathematics is meant a course in mathematics including certain elements of arithmetic, algebra, intuitive geometry, and numerical trigonometry which have been selected for their social worth or for their thinking value and which have been woven together by some basic, underlying principle into a course which is pedagogically and psychologically sound. It does not mean a hit and miss hodge-podge taught for the sake of fusion, nor does it mean a teaching of these various subjects by cycles of from three to six weeks each.

The bitter opposition to unified mathematics has been due to several reasons. The exact meaning of the term has been misunderstood by some teachers who have not taken the trouble to study the question, being content to plod along in the old

traditional path. The first movement for unified mathematics was made in grades nine to twelve and certain unsuccessful experiments have shown that this was not the place to start. With the waxing movement of the junior high school unified mathematics is coming into its own, and it is safe to predict that it is here to stay.

In grades one to six, the pupil is engaged in learning the salient principles of arithmetic. When he reaches the seventh grade there is a fork in the mathematics road. We can offer him arithmetic in the form of unmotivated reviews leading to a study of social economic arithmetic, much of which he cannot comprehend. Or, we can, by means of a unified course, give him an appreciation of the significance of mathematics beyond arithmetic, as well as certain information not contained in arithmetic which is useful to the average intelligent citizen.

Why does unified mathematics seems to be peculiarly suited to the junior high school? Let us consider the aims of the junior high school as they apply to curriculum construction. The junior high school must serve three types of pupils—those who will go on to the senior high school, those who will leave school after completing the ninth grade, and those who will drop out sometime between the seventh and ninth years. To those pupils who will not complete the junior high school, and to those who will not enter the senior high school, the junior high school must give the most valuable information and training which can be assimilated by pupils of that age. For those who will go on to the senior high school, it must reveal abilities, interests, and aptitudes, and it must bridge the gap between the elementary school and the senior high school. Applied to mathematics, this means that the junior high school must give those elements of mathematics which will prove useful to the average citizen; it must disclose mathematical ability or the lack of it; it must bridge the gap between arithmetic and the mathematics of the senior high school. Such an aim demands the best of what mathematics has to offer, and it necessitates a breaking away from the traditional course in arithmetic in these grades. We must take the best from arithmetic, the best from algebra, the best from geometry, and the best from trignometry, and weave these "bests" into a unified course which will be interesting and worth while from a sociological and mental point of view.

The problem of arranging such a course in mathematics is one of content, sequence, and method. An examination of the flood of textbooks in junior high school mathematics discloses a marked variation in them, which is strong evidence of the fact that the mathematics problem of these grades has not been satisfactorily solved. There are two aspects to the problem—the objective, that of adjusting the pupil to his environment, and the subjective, that of developing the powers of the individual. This is a problem which cannot be solved in a week or a month or a year. It may take five years more of experimentation to organize a satisfactory course. Even then, it is to be hoped that the course in mathematics in the junior high school will not be in any sense rigid or static, but that it will constantly be improved upon as a result of experimentation. The national committee says on this point (page 19): "The committee is fully aware of the widespread desire on the part of teachers throughout the country for a detailed syllabus by years or half-years which shall give the best order of topics with specific time allotments for each. This desire cannot be met at the present time for the simple reason that no one knows what is the best order of topics, nor how much time should be devoted to each in an ideal course. The committee feels that its recommendations should be so formulated as to give every encouragement to further experimentation rather than to restrict the teacher's freedom by a standardized syllabus."

In discussing the mathematics for years seven, eight and nine, the national committee states that it is of the opinion "that the material included should be required of all pupils as it includes mathematical knowledge and training which is likely to be needed by every citizen." The national committee does not say in so many words that it favors unified mathematics in the junior high school, and this was, of course, a disappointment to the advocates of unified mathematics, but it does say (page 21), "stated by topics rather than years the mathematics of grades seven, eight and nine may be properly expected to include the following: arithmetic, . . . intuitive geometry, . . . algebra, . . . numerical trigonometry, . . . demonstrative geometry, . . . history and biography." It also mentions optional topics such as the use of the slide rule, the use of logarithms and of other simple tables; it enumerates topics to be

omitted or postponed, such as cube root, the binomial theorem, etc; and it makes some suggestions on "problems," on numerical computation, and on the use of tables. The report then discusses the arrangement of this material in the light of two situations—the junior high school, and schools organized on the 8-4 basis. Five plans for the distribution of time are suggested by the committee, but no one of them is recommended as superior to the others, and only the larger divisions of the material are mentioned. In discussing the mathematics to be given in grades seven, eight and nine in schools organized on the 8-4 basis, the national report says (pages 30 and 31):

"It cannot be too strongly emphasized that, in the case of the older and at present more prevalent plan of the 8-4 school organization, the work in mathematics of the seventh, eighth and ninth grades should also be organized to include the material here suggested. The prevailing practice of devoting the seventh and eighth grades almost exclusively to the study of arithmetic is generally recognized as a wasteful marking of time. It is mainly in these years that American children fall behind their European brothers and sisters. No essentially new arithmetic principles are taught in these years, and the attempt to apply previously learned principles to new situations in the more advanced business and economic aspects of arithmetic is doomed to failure on account of the fact that the situations in question are not and cannot be made real and significant to pupils of this age. The same principles should govern the selection and arrangement of material in mathematics for the seventh and eighth grades of a grade school as govern the selection for the corresponding grades of a junior high school, with this exception: Under the 8-4 form of organization many pupils will leave school at the end of the eighth year. This fact must receive due consideration. The work of the seventh and eighth years should be so planned as to give the pupils in these grades the most valuable mathematical information and training that they are capable of receiving in these years, with little reference to the courses that they may take in later years. As to possibilities for arrangement, reference may be made to the plans given above for the first two years of the junior high school. When the work in mathematics of the seventh and eighth grades has been thus

reorganized, the work of the first year of a standard four-year high school should complete the program suggested."

The material for a course in unified mathematics should contribute to the aims of teaching mathematics as well as to the aims of the junior high school. Any course in mathematics should aim to develop the power to understand and to analyze those relations of quantity and of space which are necessary for an insight into and a control over our environment. It should make clear the importance of mathematics to modern civilization. It should furnish methods which will be effective in the life of the individual. Such a course should include the formula, graphic representation, the equation, direct and indirect measurement, computation, congruence, and similarity, with their applications.

After the material for the course has been selected, there arises the question of organization, of weaving these elements together. According to Breslich, the material should be arranged in pedagogical rather than in logical units, if lasting results are to be attained. That material should be put together which can most economically and effectively be learned together. He advocated the unit system of teaching which has been so successfully worked out at the University of Chicago. He suggested intuitive geometry as a possible means of unifying the work in the seventh and eighth grades. There are others who also advocate that intuitive geometry be taught before algebra, basing their arguments upon the belief that the mental development of the child resembles that of the race and that the development of geometry preceded that of algebra. Another argument for this arrangement is that the geometry furnishes a basis for the work in algebra when it is introduced.

Whatever may be the plan of organization, it will not give satisfaction unless, underneath it and interwoven through it, there is some basic idea, such as the function notion. Fusion itself is of minor importance as compared with this principle, and logical sequence need not be sacrificed to it. The teacher must never lose sight of this principle and she must make her pupils conscious of it. The national report says of the function idea (page 12): "The one great idea which is best adapted to unify the course is that of the functional relation. The concept of a variable and of the dependence of one variable upon another is

of fundamental importance to everyone." There are some teachers of mathematics who believe that immature pupils cannot grasp the function idea and that they should never hear the word "function" in elementary mathematics. In Europe, however, this work is begun in some schools in the sixth year of the elementary school, and in almost every school this idea is not delayed later than the seventh year. It is true that the general and more abstract form of the function concept cannot be significant to a student until he has had considerable experience in mathematics. However, by means of the tabulation of data, and of the study of the formula and graph, certain concrete examples of the variable and dependence can be presented even in the early part of the course. The primary idea of a course in unified mathematics might well be the idea of relationship between variables together with the methods of determining, expressing and representing this relationship.

After the material for the course has been selected and organized, there must be found the most effective ways of presenting it. Needless to say, the method of presentation must be in accord with the principles of the modern psychology of learning. The success of any plan of unified mathematics will rest, in the final analysis, upon the teacher. The best system will fail in the hands of a poor and an unsympathetic teacher, while a good teacher who believes in the movement for unified mathematics, will be able to make something out of a poor course. The teacher of unified mathematics must have an exceptional knowledge of her subject and of its applications so that she will be able to adapt the work to the changing interests of her pupils. She must present each phase of the work clearly, thoroughly, and gradually, emphasizing the important points, and striving to make her pupils thinkers and not mere manipulators. An unprepared or unsympathetic teacher will only bring about confusion. Neither teachers of arithmetic nor high school mathematics teachers will prove successful as teachers of unified mathematics without some special training along this line. In Rochester, New York, good grammar school teachers are selected and trained in extension courses or in Saturday institutes before they are allowed to attempt to teach the course.

What are some of the dangers of unified mathematics? The first, that of poor teaching, has been mentioned. Confusion may

easily arise in unified mathematics, especially if there is no unifying principle present. It is well nigh impossible to put good correlation into a text book. The teacher must constantly read into it the connection between the pages and the various chapters. Otherwise, the teaching of unified mathematics may resolve itself into a simultaneous teaching of arithmetic, algebra and geometry in some form not dissimilar to the present traditional courses, or it may result in the teaching of these subjects by cycles. Care must be taken not to make the course too difficult and beyond the comprehension of the pupils. Enthusiasm is very likely to carry the teacher too far in the presentation of certain aspects of the course. One should not attempt in any way to make a course in unified mathematics in the junior high school one to meet the requirements of courses in the senior high school.

What are some of the advantages of a course in unified mathematics in the junior high school over the traditional courses? In the first place, unified mathematics makes a distinct contribution to each of the three types of pupils it aims to serve. The pupil who leaves school at the end of the ninth grade, or the one who does not continue with the study of mathematics beyond this point has a better mathemaical equipment than he would have had by following the traditional course. He has a rational idea of our number system, a comprehension of the algebraic equation as a handy tool, some idea of spatial relations, the ability to use and interpret the graph, and some idea of the uses of the trigonometric functions. The pupil who leaves school before completing the ninth grade has the best and most useful mathematical knowledge which he is capable of assimilating and which could be presented to him in the length of time that he has remained in school. Unified mathematics gives to the pupil who will continue his study of mathematics in the senior high school a pre-view of these courses and a perspective of their content, nature and purpose. For him it prevents an abruptness of transition from elementary school to senior high school mathematics. A course in unified mathematics brings algebraic and geometric methods into closer correlation from the start and it furnishes a longer period in which to associate and to practice with these methods. It furnishes the individual with a greater number of modes of attacking

problems. Unified mathematics correlates readily with other school subjects.

What effect will the teaching of unified mathematics in the junior high school have upon the mathematics of the senior high school? It probably will help to determine to a large extent those who will elect mathematics in the senior high school. If it fulfills its purpose in the junior high school, unified mathematics will, in addition to this, work an improvement in the teaching of mathematics in the senior high school. It may be said that the training in mathematics which is offered by the average American high school today contributes very little to pupil or to adult needs. Much of the method of instruction is not psychological. The present courses are designed primarily for the pupil who will enter college, and they try to train him for the old-fashioned, formal type of college mathematics. If a pupil has had the advantage of a unified course in the junior high school, for him the senior high school courses should be richer, more intensive, and more enjoyable than they are at present. For example, the teacher of beginning geometry too often tries to plunge her pupils into the mysteries of demonstration in much the same way that some small boys are taught to swim, only with more disastrous results. The wise teacher works gradually into demonstrative geometry through preliminary work in intuitive geometry. Would it not save time at this point if the pupils had already been exposed to intuitive geometry in the junior high school? The time thus saved could be devoted to more intensive work than is now possible. Again, we find that about thirty per cent of the pupils registering for beginning algebra are killed off, algebraically speaking. This high mortality rate is due partly to failure to comprehend the language and symbolism of algebra, and partly to the fact that we now have a less selected group of pupils with whom to work than used to be the case. For the rank and file, one year is not a long enough time to master all the formal work we attempt to give in algebra. If pupils have been exposed to the algebraic method and language for at least two years, does it not seem reasonable to expect that they will take up the work of the more formal algebra of the senior high school with more ease and a greater degree of success? The teachers of mathematics in the senior high school, in order to hold the interest of pupils who

have been raised on unified mathematics will need to pay closer attention to the natural points of correlation between algebra and geometry. They will have to look to their methods and they will need to keep up with the best mathematical thought of the day. Senior high school courses will come to stress that which is vital, for example, the formula and the graph, even to the exclusion of such matters as the cube root of polynomials. We find that in the past teachers have voted to omit the few pages of the text devoted to he formula. It is not unusual to find that in the second semester course in geometry very few pupils can tell what a formula is or what it is good for or how to manipulate one. They fail to recognize it as an equation. As for graphic work, we barely scratch the surface of its possibilities. As for the function idea, how many pupils, even after six semesters of work in high school mathematics, have any idea of dependence? Lack of time may be offered as an excuse, but the real root of the trouble is that too many teachers are content to pass on to the present generation the sort of mathematics which they themselves studied.

It is possible and desirable to introduce the more elementary ideas of solid geometry in connection with related ideas in plane geometry. The work in solid geometry should include numerous exercises in computation based upon the formulas that are established. This will serve to correlate the work with arithmetic and with algebra. Not enough time can be given to logarithms in the third semester of algebra to insure an efficient working knowledge of them. Logarithms may be used to good advantage in working the problems of solid geometry. Elementary ideas of the calculus can well be introduced into advanced algebra and solid geometry.

It is not unlikely that the time is not far distant when unified courses in mathematics will be offered in the senior high school as electives in lieu of the college preparatory courses. The national committee on page 39 of its report says that methods of organization are being experimentally perfected whereby unified courses may be used to present those ideas which should be included in the senior high school courses. Four such plans are suggested by the committee, the central idea of unification being functionality and graphic representation.

What conclusions can be safely drawn with regard to the present status of unified mathematics? First, unified mathematics is the logical, psychological, and pedagogical solution to the problem of mathematics in the junior high school. As yet it is in the experimental stage. Several years of scientific experimentation will be necessary to determine the most valuable content and the proper method of organization. Scientific experimentation implies the testing of any given plan with ability groups of pupils under the direction of sympathetic and well-prepared teachers who will keep a systematic record of the experiment and measure their results by means of standardized tests. A few such experiments have been made. Fifteen of these are described in Chapter XII of the National Report. Second, a few experiments have been made with unified mathematics in the senior high school. The chief aim of mathematics in the senior high school will probably always be that of preparation for college. The newer movements in senior high school mathematics point to a stronger emphasis upon the natural points of correlation between algebra and geometry and upon the formula and the graph, and also to the introduction of elementary ideas of the calculus into high school mathematics. The average high school will no doubt always offer stratified courses, but it is safe to predict that unified courses will, in time, be offered in the senior high school. Third, the movement of unified mathematics has even crept up into the colleges themselves, and we find certain experimental courses for college freshmen.

Unified mathematics can be truly said to be successful or a failure only when its advocates and its foes alike have weighed fairly and open-mindedly the results of these experiments in the light of educational and social needs.

MINUTES OF CHICAGO MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, FEBRUARY 23, 1924

The meeting was called to order in the LaSalle Hotel, at 10:30 A. M. by Mr. Austin, acting in the absence of President Minnick.

The secretary read the minutes of the last meeting which were approved.

Mr. Austin explained the arrangements made by the Chicago Mathematics Club for the dinner to be given in the evening.

The treasurer's report was read and referred to the auditing committee consisting of A. H. Huntington, St. Louis; Miss Worden, Detroit; Dr. Lyttle, Urbana, Ill.

After discussion of the membership list, a committee was appointed to consider the question of combining the positions of editor and business manager of MATHEMATICS TEACHER, and to report at the afternoon session. The committee consisted of the following persons: Miss Gugle, W. D. Reeve, Mabel Sykes, Raleigh Schorling, Miss Worden, C. A. Austin, J. A. Foberg, Dr. Taylor, Miss Bixby, E. W. Schreiber.

On motion of Miss Gugle, the meeting passed a vote of appreciation of the services thus far rendered the Council in connection with THE MATHEMATICS TEACHER by Mr. Clark and Mr. Foberg.

The mornnig session closed with the reading of a paper on "A Better Use of Tests," by W. D. Reeve, Teachers College.

At the afternoon session, Mr. Huntington, St. Louis, reported for the Auditing Committee that the Treasurer's books were in agreement and showed a balance in the bank of $735.64. Certain amounts due the publisher are outstanding, for which bills have not yet been rendered.

Miss Gugle reported for the Committee of Ten appointed at the morning session as follows:

The committee recommends to the National Council that the Executive Committee be authorized and directed to enter into legal contract with Mr. John R. Clark by which he agrees to edit and manage MATHEMATICS TEACHER for a period of two years on the following conditions:

1. The subscription price of MATHEMATICS TEACHER including membership in the Council is to be two dollars, of which the editor-manager shall receive one dollar seventy-five cents and the Council shall receive twenty-five cents.

2. Each number of MATHEMATICS TEACHER shall contain a minimum of 62 pages of printed matter exclusive of advertisements.

3. There shall be eight issues of the magazine each year.

4. The proprietary interest in MATHEMATICS TEACHER shall continue to rest in the National Council of Teachers of Mathematics.

On motion, this report of the committee was adopted.

The Executive Committee recommended that a considerable part of the next annual meeting be concerned with research on problems in the teaching of mathematics in the high school, such as:

1. Tests in the junior high school.
2. Objectives for nine year algebra.
3. Degree of mastery in mathematics that the pupils have when they come to the study of science.
4. The I. Q. that is necessary for satisfactory work in plane geometry.

It is understood that members of the National Council shall contribute to this work.

Mr. Schreiber moved that the Executive Committee be ordered to prepare a complete record of the membership of the Council to be printed as a part of the magazine or as a bulletin, the list to be geographical.

On motion, it was decided to fix the date of the next annual meeting at the Saturday preceding the week of the meeting of the Department of Superintendence of the N. E. A. for the year 1925.

Miss Gugle reported as follows for the Nominating Committee:

For President—Raleigh Schorling, School of Education, University of Michigan.

Vice President—Miss Florence Bixby, Riverside High School, Milwaukee.

Executive Committee—Miss Orpha Worden, Teachers College of Detroit, and C. A. Austin, Oak Park, Ill.

On motion, the Secretary was directed to cast the ballot of the Council for these nominees, and they were declared elected.

The Secretary was directed to send a telegram of greeting to Dr. Minnick, who was kept from the meeting by illness.

The following papers were read at the afternoon session: "Memory and Marks in Mathematics," Ethel Luccock, Northwestern High School, Detroit; "The Laboratory Method in the Class Room," Charles Stone, Chicago University High School.

At 6:50 p. m. a very enjoyable banquet, arranged by the Chicago Mathematical Club, was participated in by a large assemblage. The program, presided over by Professor H. E. Slaught, consisted of the following papers: "Reliability of Teachers' Marks," Raleigh Schorling, Michigan University High School, Ann Arbor, Michigan; "Teaching Pupils the Conscious Use of a Technique of Thinking," Elsie P. Johnson, Oak Park and River Forest Township High School, Oak Park, Ill.

The following people registered at the meeting:

Allen, J. Eli, Philips H. S., Birmingham, Ala.
Arends, A. T., Peoria, Ill.
Bawhill, Jno. F., Michigan State Normal, Ypsilanti, Mich.
Beyer, Althea U., Elgin H. S., Elgin, Ill.
Broslick, University of Chicago, Chicago, Ill.
Campbell, Sadie, Fort Smith, Ark.
Carroll, Nancy G., New Trier H. S., Kenilworth, Ill.
Carter, Helen L., Elgin H. S., Elgin, Ill.
Cobb, Agnes, Harrison Tech. H. S., Chicago, Ill.
Conner, J. A., Harrison Tech. H. S., Chicago, Ill.
Coons, Mrs. G. O., Riverside H. S., Milwaukee, Wis.
Coultrap, W. W., Northwestern College, Naperville, Ill.
Croppen, Grace, Evanston Twp. H. S., Evanston, Ill.
Crewes, Frances, Riverside H. S., Milwaukee, Wis.
Cowles, Mrs. Jean, Madison Central H. S., Madison, Wis.
Davis, James E., Central Prep. Schools, Chicago, Ill.
Fisher, Marian B., Elgin H. S., Elgin, Ill.
Foberg, J. A., State Dept. Public Instruction, Harrisburg, Pa.
Gorsline, W. W., Crane Junior College, Chicago, Ill.
Hacker, J. A., 6140 University Ave., Chicago, Ill.; Student U. of C.
Harper, G. A., New Trier H. S., Kenilworth, Ill.
Herr, Ross, Chicago Normal School, Chicago, Ill.
Hester, F. O., Lane Technical H. S., Chicago, Ill.
Hicks, E. L., Lake View H. S., Chicago, Ill.
Hinkle, E. C., Chicago Normal School, Chicago, Ill.
Hudson, Joseph, Lake View H. S., Chicago, Ill.
Johnson, A. T., Chicago Normal School, Chicago, Ill.
Johnson, Elsie Parker, Oak Park H. S., Oak Park, Ill.
Johnson, Stella M., Tilden H. S., Chicago, Ill.
Jones, C. Herbert, New Trier H. S., Kenilworth, Ill.
Kaher, Marie H., Lane Technical H. S., Chicago, Ill.
Kaliler, F. A., New Trier H. S., Kenilworth, Ill.

Kesler, L. P., Eureka, Ill.
Kinney, J. M., Crane College, Chicago, Ill.
Laughlin, B., Chicago Normal College, Chicago, Ill.
Leach, E. S., Evanston Twp. H. S., Evanston, Ill.
Lindner, Mildred D., J. Sterling Morton High, Cicero, Ill.
Leonard, Sylvia, Riverside H. S., Milwaukee, Wis.
Luccock, Ethel, Northwestern H. S., Detroit, Mich.
Luccock, Natalie, Deb. Teachers College, Detroit, Mich.
Lundquist, Hilden, Hyde Park H. S., Chicago, Ill.
McCain, A. B., Washington H. S., Milwaukee, Wis.
McMiechael, Ellen M., 6110 Dorchester St., Chicago, Ill.
Marks, F. R., Lewis Inst., Chicago, Ill.
Marr, Roy T. Marr, Central Y. M. C. A. Prep. Schools, Chicago, Ill.
Moore, E. L., 2138 W. 107th Place, Chicago, Ill.
Newell, M. J., Evanston H. S., Evanston, Ill.
Nuttale, J. T., Crane H. S., Chicago, Ill.
Nyberg, Jos A., Hyde Park H. S., Chicago, Ill.
Olson, Emma J., 6030 Greenwood Ave., Chicago, Ill.
Palmer, C. E., Glenbud H. S., Glen Ellyn, Ill.
Pierce, P. R., 953 N. Pine Ave., Chicago, Ill.
Pope, Walter S., Morton H. S., Cicero, Ill.
Pratt, Adah, Elgin H. S., Elgin, Ill.
Price, Lellis, New Trier H. S., Kenilworth, Ill.
Prutsman, Eunice M., J. Sterling Morton High, Cicero, Ill.
Rebbe, A., Thornton Twp. H. S., Harvey, Ill.
Reeve, W. D., Teachers College, New York City.
Richards, Helen M., 43 N. Minard Ave., Chicago, Ill.
Ringold, Inez L., New Trier H. S., Kenilworth, Ill.
Roberts, Louise A., Austin H. S., Chicago, Ill.
Samuelson, Verna R., Elgin H. S., Elgin, Ill.
Schaeffer, A. F., Waukegan H. S., Waukegan, Ill.
Schorling, Raleigh, University of Michigan, Ann Arbor, Mich.
Schreiber, Edwin W., Proviso, Maywood, Ill.
Searse, William A., Lane Tech, Chicago, Ill.
Slaight, Chester, Greenville, Mich.
Slaught, H. E., University of Chicago, Chicago, Ill.
Smith, Elsie, Harrison Tech. H. S., Chicago, Ill.
Stokes, C. U., New Trier H. S., Kenilworth, Ill.
Stone, C. A., N. High, Chicago, Ill.
Taylor, E. H., Eastern Illinois Teachers College, Charlestown, Ill.
Thompson, Mary E., Morgan Park H. S., Chicago, Ill.
Vanderpol, Tanetta, Lane Tech H., Chicago, Ill.
Werremeyer, D. W., West Tech. H. S., Cleveland, Ohio; (Delegate Cleveland Mathematics Club)
Whitaker, J. I., Batavia H. S., Chicago, Ill.
Williams, Mary Louise, Kenosha H. S., Kenosha, Wis.
Woessner, Anna L., Tilden Tech., Chicago, Ill.
Wright, H. C., Deerfield Shields H. S., Highland Park, Ill.

Delegates representing organizations:

Mathematics Club of Cleveland, D. W. Werremeyer; Mathematics Club of Columbus, Ohio, Gertrude Silver, North H. S., Columbus, and Amy Preston, Roosevelt Junior H. S.; Mathematics Club of Detroit, Ethel Luccock; Mathematics Club of St. Louis and vicinity, Albert H. Huntington.

NEWS NOTES

THE University of Chicago Conference. With Chicago's assistant superintendent, Mr. William J. Bogan, as chairman, the mathematics section of the University of Chicago Conference was called to order in Reynolds Club Theatre, Friday afternoon, May 9th.

Only those who know Dr. Slaught of the University of Chicago, and his delightful way of telling a story can appreciate "the secret society of numbers" which was his way of treating his topic, "Mathematical References in the Teaching of Secondary Mathematics." From a very loosely organized affair in which there was no form, Dr. Slaught told us, numbers acquired symbolic representation and position from the Eastern Ancients. After the admittance of the new members, negative numbers, vouched for by Des Cartes in the seventeenth century, the society was reorganized with a constitution telling the business of numbers: namely $+$, $-$, $\times$, $\div$. It also had by-laws, those laws which we know by the names distributive, associative, and cumulative.

Next came fractions asking for membership which was granted them after they were proved to obey these laws. That situation, however, created the necessity of an amendment to the constitution,—the prohibition amendment, if you please—that 0 cannot be used as a divisor.

Later came the surds and finally our so-called imaginary numbers to complete the membership of this secret society. And so we have the history of number.

In the second paper of the afternoon, Mr. J. O. Pyle of the Harrison Technical High School, Chicago, urged very earnestly that we approach the study of secondary mathematics as the natural sciences are approached. He prophesied in that case a vast improvement in the attitude of both teacher and taught as well as in the method of teaching.

Then came the event for which everyone was expectantly waiting, Mr. Ernst R. Breslich's demonstration class in supervised study.

The class consisted of thirty pupils of the University High

School, University of Chicago, where Mr. Breslich is head of the Mathematics Department. Part of these thirty pupils were from the junior high school and part from the regular third year class, but all are studying Mr. Breslich's "Third Year Mathematics."

The boys and girls who, Mr. Breslich said, averaged about sixteen years of age, came in quietly and, as is their custom, set at once to work, solving triangles by the sine law. There is no home work, though each pupil is urged to review the work of the class and the better pupils have outside projects on which they work when they have outstripped the ones who move more slowly. But for the most part work goes on from day to day as it went on that afternoon. After a little time, they were asked to study for themselves the exposition in their text book of the tangent law. Ten minutes was spent in study after which one of the students who had finished was called on to explain the law to the class. A brilliant girl, she gave a wonderfully clear and concise explanation of that very difficult law. Contrary to his usual custom, due to lack of time, Mr. Breslich gave his class no opportunity to ask questions, but after the solution of several examples, they went on to solve problems involving the new formula.

Much discussion was provoked by this interesting demonstration but had to be cut short because of the lateness of the hour.

Mr. E. W. Schreiber, Provise Township High School, Maywood, Ill., as chairman reported that the nominating committee had chosen as the new member of the conference committee, Mr. Joseph Nyberg, Hyde Park High School, Chicago. After accepting the report of the committee, the Section adjourned. —Gertrude L. Anthony, Oak Park High School, Oak Park, Ill.

THE fourth annual meeting of the Inland Empire Council of Teachers of Mathematics was held at the Lewis and Clark High School, Spokane, Wednesday and Thursday afternoons, April 9th and 10th, 1924. The meeting was called to order by the President, W. W. Jones, of North Central High School, Spokane. The members for the year 1923 answered to roll call.

Prof. N. J. Lennes of the University of Montana gave an interesting talk on "Modern Psychology and the Teaching of

Algebra.'' Three important uses of psychology in the teaching of algebra were stressed; first disassociation of ideas taught, for instance, if we strive to teach neatness, to be of any value to the student, he must learn to connect with other subjects as well as his algebra paper; second, reviews should be spaced with due regard to the meaning involved, definitions should carry over a meaning and not merely a group of words. An interesting discussion, led by Jessie Oldt of the North Central High School of Spokane, followed.

The next address of the afternoon was given by Prof. Eugene Taylor of the University of Idaho on ''Mathematical Experience.'' In this, he compared our mathematical experiences with experiences in other lines of activity such as religion or music. The council is much indebted to Prof. Taylor for a very carefully prepared paper.

The last address of the afternoon was given by Leona Coulter of the Lewis and Clark High School, Spokane. In this she reviewed Robinson's *The Mind in the Making*. This was one of the books on a list prepared last year as a suggested reading course for mathematics teachers. The review was most interesting and inspired those present with a desire to do more of the reading suggested.

The President then appointed the nominating committee consisting of W. C. Eells, Olive G. Fisher, and Eugene Taylor, to report the following afternoon.

The second session was called to order by President Jones. The minutes of the previous year were read and approved. A motion was made an seconded that dues for the current year be waived since there was a balance of $10.50 in the treasury. Motion carried. The report of the nominating committee was then read by Chairman Eells: for President, N. J. Lennes, of the University of Montana; Secretary, Christine Claussen, Lewis and Clark High School, Spokane; Member of Executive Committee for three years, J. C. Teeters, Kellogg, Idaho. Moved and seconded that the report of the committee be accepted. Motion carried.

The first number on the program of this afternoon was an address by Prof. I. C. Isaacs of the Washington State College on ''The Algebra of the Infinite Number.'' This talk aroused much discussion among the members present as to how much

should be taught in high school about the infinite number and the best method of presenting it so that the students will not acquire an erroneous idea of that number.

Anna Whitney of Yakima then gave an interesting talk on "General Mathematics," showing how it is being used in her school to benefit the weaker student. The members of the council were much interested for Miss Whitney could bring much in the way of her personal experience.

The last discussion of the afternoon was given by Prof. Walter C. Eells of Whitman College, Walla Walla, on "Standard Mathematical Tests." Prof. Eells gave suggestions as to the tests of the various branches of mathematics; arithmetic, algebra, geometry and general mathematical ability. He explained the various good and bad points in tests and urged the greatest of caution in using them. Following a lively discussion a motion was made and seconded that a committee with Mr. Eells as chairman be appointed to give these tests in the various schools of the Inland Empire and to act as a clearing house for the results. Motion carried.

Motion made and seconded that fourth annual meeting be adjourned to meet the following year. Motion carried.

CHRISTINE CLAUSSEN, *Secretary.*

Lewis and Clark High School,
Spokane, Wash.

NEW BOOKS

The Rhind Papyrus. By T. ERIC PEET. London. Hodder & Stoughton. 1923. 135 pp. + XXI plates. Price, 3 guineas.

It will be a source of much interest to teachers of mathematics to know that a new edition of the famous *Ahmes Papyrus* has just appeared under the editorship of Professor Peet, of the University of Liverpool, one of the leading Egyptologists of the present time. He has given a new translation of the entire work, and besides that has added in the plate a transcription of the work from the original hieratic into hieroglyphic. We thus have a scholarly English edition of the oldest extant manuscript of an extensive mathematical treatise in the world, a work written as early as 1550 B. C., and probably somewhat earlier.

It will be remembered that the work was translated by Professor Eisenlohr more than fifty years ago. This translation was not entirely satisfactory in many respects and the historical information was not as complete as it is possible now to give.

Professor Peet has apparently made a very careful tranlation, and he has added a large number of helpful notes. Although the price of the book makes it probable that few teachers will feel like purchasing it for their own private libraries, nevertheless it is a work that should be in all school and college libraries where any attention is paid to the historical development of the mathematics taught.

DAVID EUGENE SMITH.

Junior High School Mathematics. By WALTER W. HART. D. C. Heath and Company, 1921.

This is a three-book series. Book One consists of 224 pages, dividing this space about evenly between arithmetic and intuitive geometry. Book Two contains 250 pages. There is a short chapter on short methods of computing, another on the technique of formulae, about 100 pages of business arithmetic, and the rest of the book is devoted to measurements, operations with positive and negative numbers, and indirect measurements by graphic methods. Book Three, about 500 pages, with the exception of a nine-page chapter on the trigonometry of right triangles, is an algebra.

In the author's preface to Book One he says (I quote in part): "The following principles have been observed in the preparation of it.

"I. *The course must be practical.* In grade seven, this means that:

"1. There must be continued drill upon the fundamentals (of arithmetic).

"2. The instruction on new topics must be limited to processes and applications encountered by the average person."

"II. *The course must be enriched.* . . . This cannot be done sufficiently by the use of problem material, local or otherwise, grouped about some central theme, as is so often attempted. On the other hand, the beginnings of literal arithmetic and inductive geometry will accomplish this purpose. . . . This use of the formula is part of a thoroughly considered plan which is developed in subsequent books. . . . The (geometry) instruction in this text is centered around the mensuration facts connected with the common plane figures and the rectangular parallelopiped, these being the practical parts of geometry for most people. More than usual attention has been given to developing the geometrical concepts involved by drawing, measuring, and other informal exercises.

"III. *The course must be thorough.* . . . Observe that the instruction is given in simple language and quite fully."

"IV. *The course must begin the process of rationalizing mathematics.* . . . A strong argument may be made for the proposition that such rationalizing in the junior high school shall be chiefly inductive and, in the senior high school, chiefly inductive.

"V. *The course must prepare for later courses.* . . . The articulation (between elementary and high schools) will be satisfactory only if the pupils acquire in the seventh and eighth grades certain correct ideas and mathematical habits and an interest in mathematics. . . .

"VI. *The course must be flexible.* All schools do not wish and cannot use the same material. . . . Certain sections, marked supplementary, are recommended for omission if it becomes necessary to save time somewhere."

The above principles are again set forth in the preface to Book Two.

With repect to mathematical content and its organization this series of books is characterized by consistency and continuity. As to *manner of presentation,* it is easy to criticise adversely. The reader cannot escape the feeling that the author is thinking too anxiously about the little details of the subject. The chapters in algebra and geometry tend to be too complete; the mathematician finds it difficult to leave ideas partially defined—to leave something for the pupil to fill in later if he should continue mathematics. For example, in Book I (page 113), under the section heading "Straight Lines," we have: "Place on paper a point 0. Through it draw a horizontal line, with arrowheads at its *ends* (italics mine). . . . The arrowheads mean that the line extends infinitely far in each of two directions. . . . Through point 0 . . . draw a vertical line, *terminated* by arrowheads." Now the purpose of this exercise is to define the terms *vertical, horizontal* and *oblique;* it is unfortunate that the author should cloud the issue by stopping to emphasize the infinitude of these lines—especially when they are said to have *ends* and to be *terminated.* There is such a thing as clarity of ideas without completeness—and completeness comes with time and experience, rather than with untimely word statements.

The influence of the older ideas of textbook making is seen also in the writer's use of rules. There are over a hundred rules in the three books, 64 in Book in Three; these rules range from 2 to 23 lines in length, the median length (Book Three) being 6 lines. While there is no objection to rules, *as such,* it is necessary to distinguish between a rule *of procedure* and the *word statement* of it. Often the statement is harder to understand than the procedure. It would seem that the author has placed undue confidence in rules as aids to learning; at any rate he has used them excessively as compared with other writers of such books.

While these books provide for much activity on the part of the pupils, they often fail to make a situation the impelling motive for such activity. The process of learning is made more interesting and the results of study more permanent when the pupil understands at the outset the general aim of a piece of work and is conscious at the end that this aim has been accomplished.

In these days of experimentation with junior high school courses and books, teachers and textbook makers can well afford to avoid carping criticism of each other's work. We all have much to learn, and we are going to learn by trying out things. Mr. Hart's methods of presentation are eminently worth trying; and as regards content and organization he has avoided most of the pitfalls into which some writers in this field have fallen.

WILLIAM F. ROANTREE.

Modern Mathematics. By RALEIGH SCHORLING and JOHN R. CLARK. World Book Company, Yonkers, New York.

One of the outstanding ideas in modern educational theory is that the best preparation for future mental growth in children is to be found in activities which give the children immediate satisfaction. It is possible to organize instruction in such a manner as to give the pupils a definite sense of desirable objectives and a conscious realization of their attainment.

In the field of junior high school mathematics the problem *has* been—*What* shall we teach, and how shall the *subject matter* be organized? While there is nothing like complete agreement on this question, it is clear that textbook writers are finding a substantial basis for agreement in the recommendations of the National Committee on Mathematical Requirements. The present lack of teachers with training and experience in junior high school work makes it necessary for textbook writers to give special attention to matters of method. A good text in junior high school mathematics must be a good guide to the teacher as well as to the pupil. On this point *Modern Mathematics* makes a strong case for itslf. In the authors' preface to the seventh year book they say, "This textbook (while in the form of *experimental editions*) has undergone three revisions. . . . The authors have been desirous of preparing a textbook which fits children's capacities, interests, and needs to a degree that would have been unattainable without the long, tedious, painstaking, experimental development.

"More than one hundred and fifty teachers, chosen to represent all degrees of teaching ability, have taught these materials before publication. In many of our cooperating schools, the children were classified according to ability. . . . Experience with widely different social and racial groups . . . in

thickly populated districts was absolutely essential for solving language difficulties. The material is written not only *for* the pupil but *to* the pupil. There is a fundamental difference between ability to understand mathematical relations and the ability to comprehend the language in which these notions are ordinarily expressed. . . . Mathematical principles are here stated in terms that children use."

Modern Mathematics is presented in the common three-book form, a book for each of the grades seven, eight and nine. The seventh year book treats of practical measurements, graphs, simple acocunts and business forms. To secure the desired skill arithmetical computation twenty-two timed practice tests are distributed throughout the book.

The eighth year book introduces the children to various methods of expressing relations between numbers, to the use of the equation, to indirect measurements by means of scale drawings and the trigonometry of the right triangle, to a practical knowledge of investments, and to the use of signed numbers.

The ninth year book is chiefly concerned with algebra, carrying the subject through quadratic equations with one unknown. A supplement contains chapters on "What Every One Should Know About Statistics," "Logarithms and Approximate Numbers," "The Slide Rule," and "The Measurement of Angles." Of this supplement the authors say: "The following pages probably contain the most valuable and interesting materials of this book."

To the teacher who is accustomed to the traditional organization of subject matter these books will seem to lack unity and coherence. One cannot turn to one chapter and find therein all that is taught about a given subject; he would have to search all three books diligently to run some of the topics down. The chapter headings reveal the principle which governs the organization of the material. To illustrate: "How to Express Relations Between Numbers," "How to Use Equations," "The Secret of Thrift," "Making Money Earn Money," "How Numbers Change Together," such topics are not mathematical unities, they do not present the static perfection of a finished science, but they "challenge the pupil," they are "thought-provoking." The *uses* of mathematics, rather than its science, gives whatever of unity is achieved. "The hope rests in what the

pupil does rather than what is told him. . . . The needs of the pupil as he attempts to solve the problem in hand determine the selection of materials.''

There are two chief aims in the teaching of junior high school mathematics, (1) to help the pupil to understand *in a mathematical sense* the more important facts and phenomena of his physical and social environment, and (2) to lay a foundation for later work in mathematics. Such a course must be a self-contained unit, must yield real returns for the investment of time and effort, and should develop a sense for mathematical relationships. ''Modern Mathematics'' is plainly written with these ends in view and may reasonably be expected to attain them.

WILLIAM F. ROANTREE.

A Companion to Elementary School Mathematics. By F. C. BOON, principal mathematical master at Dulwich College, England. London, Longmans, Green & Co., 1924. Pp. 302. Price, 14 shillings.

This is the most recent book of Longmans' Modern Mathematical Series, a series comprising such worthy and well-known works as those of Professors Nunn, Carslaw, Carey, Bateman, and Mathews, Miss Punnett, Mr Abbott, the late Mr. C. S. Jackson, and various others. It therefore has as its companions a number of modern and progressive studies in mathematical teaching.

It is not, however, a book on pedagogy (if we must continue to use the term), although it is pre-eminently a book for teachers. It offers the kind type of knowledge that supplements that which comes from any ordinary college course—knowledge which the modern teacher needs but is unable to find in the textbooks that he uses in his classroom or has studied in his elementary work. After two chapters devoted to the biographies of eminent mathematicians and to a sketch of the history of the elementary topics taught in the schools, the author takes up the following topics: Euclid's postulates and the ''three famous problems''; the squaring of the circle and the development of methods for approximating π; certain well-known curves and their applications; the theorems of Pythagoras, with various methods of attack; the symbols and nomenclature of mathe-

matics; symmetry in planes and in space; analogy—a very convenient term, but one which we in America more commonly absorb in such terms as generalization, correlation of algebra and geometry, and correlation of plane and solid geometry; degree; continuity, treated more broadly than with us; negative magnitudes—that is, the negative applied to geometry; complex numbers; limits; converses; inequalities; induction; paradoxes and fallacies, besides various other topics of the same general type.

Personally, this reviewer feels that the biographical and historical material is not worthy of ranking with the rest of the book, and that there should have been some cold-blooded critic to revise it before it was published. This, however, is a minor matter; the rest of the book contains some good mathematics, it presents a large number of things that the teacher should know and that will help him in modernizing his work, and it will assist in getting us all out of the ruts which tradition tends to build up about us. It is with no little regret that it must be said that the ideas can be used more successfully in the schools of England than in those of America. Whether the progress of democratic education there will bring the level of achievement as low as it is with us is a problem for the future to determine. With our growing appreciation of individual differences, and with our fortunate beginning in the recognition of the rights of the more capable student, it seems probable that the outcome will be that, a generation or two hence, the standards will be about the same in the two countries.

DAVID EUGENE SMITH.

Modern Algebra. Ninth School Year. By RALEIGH SCHORLING and JOHN R. CLARK. World Book Company. 1924.

Milne-Downey First Year Algebra. By WILLIAM J. MILNE and WALTER F. DOWNEY. American Book Company. 1924.

First Course in Algebra. By JOSEPH A. NYBERG. American Book Company. 1924.

The preface in each of these three books begins with a refer-

* The Reorganization of Mathematics in Secondary Education: A Report by the National Committee on Mathematical Requirements. 1923.

ence to the National Committee Report of 1923*, and in all three of them many of the recommendations of that report are faithfully carried out. The formula and the graph are studied early and very thoroughly; ideas of geometry are utilized, sine cosine and tangent are defined and used, significant figures are at least mentioned, and the idea of function, if not the name, is brought into the work in detailed expositions. Solutions are systematically checked. The topics to be omitted or postponed are out. There is a great deal to be thankful for.

Trigonometry, of right triangles only, appears in due obedience to the suggestion of the Report and to the Requirements of the College Entrance Board. It comes at the end, however, has little to do with algebra, and must appear to the student as an additional and unrelated subject. It seems reasonable to hope that some day it will be incorporated with the geometry, where it does of right belong.

In all three of the books, there is careful preparatory discussion of the difficulties in forming the equations for "verbal" problems, and for handling the equations when they are obtained (or proposed). Schorling and Clark, and Nyberg, cling to the notion that the "unknown number" should be chased to the left side of the equation; Milne and Downey treat both sides impartially. All of them ignore the significant fact that "completing the square," being a reduction to the form $z^2=k^2$, can be treated as a direct and unfailing method of factoring.

As between the orthodox method for square root (of numbers) and the "estimate and average" method*, the National Committee report prudently refrains from any recommendation; accordingly we find that Nyberg gives only the orthodox method, Milne and Downey give the orthodox method with the estimate method as an alternative, Schorling and Clark give the estimate method and omit the orthodox method. Not one of the books comments on the rapidity of approximation by either method.

The National Committee Report lays great emphasis on the recommendation that accuracy and flexibility in numerical computation ("of vital significance") be fostered by effective drill throughout the secondary school period, and that among the aims of greatest importance should be the exercise of common sense and judgment in computing from approximate data.

Work of this kind is not really plentiful in any one of the books; Nyberg does not mention the subject until towards the end, Schorling and Clerk discuss it also at the end, Milne and Downey deal with it early and use it occasionally.

Multiplying from the left-hand end of the multiplier ("reverse multiplication") I do not find in Nyberg, or in Milne and Downey, but these authors may assume the teaching of it in earlier grades; Schorling and Clark have a curious substitute for it which will require some careful teaching to prevent an impression of artifice. In view of the fact that this custom which seems so novel is really not very new, has won some recognition in this country, and is recommended most emphatically in the kindred English report*, any other device for the same purpose should be of conspicuous merit.

GEORGE W. EVANS.

Charlestown High School,
Boston, Mass.

Essentials of Algebra. By DAVID EUGENE SMITH and WILLIAM DAVID REEVE. Ginn and Company, 1924.

Most authors of recent texts in algebra have shown a decided tendency to reduce very markedly the emphasis that has been given to special products, factoring, fractions and operations with long polynomials. The formula and the graph, along with the notion of *dependence,* have been more fully treated. Numerical trigonometry has been given a place in first-year texts. Timed-practice tests (with definite norms) have replaced the indiscriminate and unmotivated practice of the older books. The value of the subject is made more apparent to the pupil. In each of these respects, *Essentials of Algebra* is conspicuously modern.

* The Reorganization of Mathematics in Secondary Education: A Report by the National Committee on Mathematical Requirements. 1923.